Professor H. T. Cory,

With compliments of

J. H. B.

NOTES

ON THE

DESIGN OF MACHINE ELEMENTS

FOR USE IN CONNECTION WITH UNWIN'S
MACHINE DESIGN, PART I.

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PREFACE.

These notes were prepared to accompany Professor W. C. Unwin's Elements of Machine Design, Part I. Taken in connection with this text-book, they form an outline of the course in the Design of Machine Elements as given to the Junior class of Sibley College, Cornell University.

The arrangement of the topics indicates the order in which the subjects are discussed, so that these notes serve as a syllabus of the lectures as well as a commentary on the text-book. In order that this double function may be fulfilled, numerous headings of articles are inserted accompanied simply by references to Professor Unwin's book. When it has seemed desirable to supplement or qualify the statements of the text-book, comments follow the appropriate references. The treatment of certain topics is quite independent of the text-book, and references to the authorities used are generally given in connection with the discussion of such subjects.

A short list of Reference Books is added to suggest the sources of fuller information and data. These books are arranged in classes, as an indication of their general scope, but they overlap to a considerable degree.

The preparation of these Notes has extended over a period of two or three years, advanced sheets having been printed and distributed to the classes from time to time. The conditions under which they have been issued has necessarily resulted in errors and imperfections, and many of these are apparent.

I desire to acknowledge my great obligation to the numerous writers and investigators consulted. I am especially indebted to Professor Dexter S. Kimball and Mr. William N. Barnard for their helpful criticism and careful reading of the manuscript and proof.

JOHN H. BARR.

Ithaca, New York,
March, 1901.

REFERENCE BOOKS.

Materials of Engineering.

Materials of Construction	R. H. Thurston
Materials of Construction	J. B. Johnson

Mechanics of Engineering.

Mechanics of Engineering	I. P. Church
Mechanics of Materials	Mansfield Merriman
Applied Mechanics	J. H. Cotterill
Mechanics of Machinery	A. B. W. Kennedy
Machinery and Millwork	W. J. M. Rankine

General Design.

The Constructor	F. Reuleaux
Machine Design	A. W. Smith
Machine Design	J. F. Klein
Machine Design	F. R. Jones

Special Subjects.

Friction and Lost Work	R. H. Thurston
Machinery of Transmission	J. Weisbach
Gearing	Brown & Sharp Mfg. Co. (Beale)
Kinematics, or Mechanical Movements	C. W. MacCord
Teeth of Gears	George B. Grant
Rope Driving	J. J. Flather

Practice.

Mechanical Engineer's Pocket-Book	Wm. Kent
Mechanical Engineer's Pocket-Book	D. R. Low
Manual of Machine Construction	John Richards

Transactions of American Society of Mechanical Engineers.
Transactions of American Society of Civil Engineers.
Transactions of American Institute of Electrical Engineers.
Transactions of American Institute of Mining Engineers.
Transactions of Institution of Civil Engineers (Great Britain).
Transactions of Institution of Mechanical Engineers (Great Britain).
Engineering Periodicals.
Trade Publications.

I.

STRAINING ACTIONS IN MACHINES.

1. **Forces acting on Machine Members.**—[Unwin, § 16, page 22.]

To the forces specified by Unwin, may be added: (7) Magnetic attraction, as exerted between members of electrical machines.

2. **Nature of Straining Actions.**—The character of the straining action and of the stress which results from a given load depends upon the direction and point of application of the load force, (or forces), and upon the form, the position, and the arrangement of the supports, of the member. A given load may produce tension, compression, shearing, flexure, or torsion; or a combination of these. Of course tension and compression cannot both exist at the same time between any pair of molecules, or particles. Flexure is a combination of tensile and compressive stresses between different sets of molecules; or, as it is often expressed, in different fibres, of the same body. Torsion is a special form of shearing stress. Owing to the frequent occurrence of flexure and torsion it is convenient to treat these as elementary forms of stress.

The stresses due to tension, compression and flexure are essentially molecular actions normal to the planes separating adjacent sets of interacting molecules: that is, the stresses increase or decrease the distances between these molecules along lines connecting them.

The primary straining effect in shearing and torsional actions is displacement of adjacent molecules, between which the stress acts, tangentially to the planes separating such molecules. In uniform shear, the interacting molecules move, relatively, with a rectilinear translation. In torsional action, the adjacent molecules, each side of the plane of stress, have a relative rotation about an axis.

3. **Ultimate or Breaking Strength.**—[Unwin, § 17, pages 23–24; also Table I, pages 40–41.] See, also, the table given on

AVERAGE VALUES OF STRENGTH OF MATERIALS.[1]

Material.	Per Cent. Carbon	Stress at Elasticity Limit. Tension.	Stress Ultimate Tension.	Stress at Elasticity Limit. Comp'n.	Stress Ultimate Comp'n.	Stress Ultimate Shearing.	Modulus of Elasticity. Shearing.	Modulus of Elasticity. Tension.
Bessemer and Siemens-Martin Steel.	0.15	42,000	63,000	39,000		48,000	9,000,000	30,000,000
	0.20	47,000	68,000	43,000		53,000		
	0.50	48,000	80,000	46,000		57,000		
	0.70	53,000	89,000	53,000		60,000		
	0.80	57,000	103,000	63,000		68,000		
	0.96	69,000	118,000	71,000		83,000		
High grade wrought iron		28,000	56,000	28,000		40,000	9,000,000	28,000,000
Common wrought iron		22,000	40,000	22,000		32,000	9,000,000	28,000,000
Crucible or tool steel		58,000	116,000	58,000			12,400,000	31,000,000
Malleable cast iron			35,000					
Steel castings		29,000	47,000	29,000				19,000,000 to 31,000,000 Average 25,000,000
Cast iron			10,000 to 35,000 Average 20,000		56,000 to 145,000 Average 90,000	18,000 to 20,000	7,000,000	13,000,000

[1] Taken from Constructive Materials of Engineering, A. W. Smith.

page 2; taken from Professor A. W. Smith's Constructive Materials of Engineering.

4. General Idea of the Factor of Safety.—[Unwin, § 17, pages 23–24.]

The working stress in a member must be less than the ultimate strength of the material, because:

(a) Members of structures and machines are not made to be broken in ordinary service.

(b) Materials employed in engineering usually take a permanent deformation, or set, before rupture occurs.

(c) There is always liability of defects in the material and imperfections in workmanship.

(d) In many cases there is danger of stress greater than the normal working stress from an occasional excess of load, or from accidents which are not foreseen or computed in advance of their occurrence.

It is generally essential that a part be not only strong enough to avoid breaking under the regular maximum working load, but also that it shall not receive a permanent set; for a machine member ordinarily becomes useless if it takes such set after having been given the required form. In many cases a temporary strain, even considerably below the elastic limit, would seriously impair the accuracy of operation, and in such cases the members often require great excess of strength to secure sufficient rigidity. It follows from these considerations that the working stress should always be below the elastic limit and it must often be much lower than the elastic strength.

The elastic strength of many of the common materials of construction is not much above one-half the ultimate strength, and the proper allowance for defects, overloading and other contingencies depends upon the conditions of the particular case. It thus appears that the working stress should never be as great as one-half, and it should seldom exceed one-third, of the working strength of the material. In structures liable to little variation of load and to no shock, the working stress may be from one-third to one fourth the ultimate strength, with such comparatively homogeneous and ductile materials as wrought iron, mild steel,

etc. With brittle materials, as cast iron, hard steel, etc. (which are more subject to hidden defects and are less reliable generally), a greater margin is required for safety. If the conditions are such that the material is apt to deteriorate seriously, a suitable decrease of computed working stress should be made.

The effect of a suddenly applied load (shock or impact) is to produce a stress in excess of that due to the same load applied gradually, and where such impulsive application of the load is to be expected, an appropriate reduction of the ordinary working stress should be made to provide for this action. Experience and experiment have shown that the repeated variation or reversal of stress affects the endurance of a material, sometimes causing a piece to break under a load which it has often previously sustained. The theory of this gradual deterioration is not very completely developed as yet; but enough has been learned to show that the working stress must be reduced as the magnitude of the variations of stress and the number of such variations increases.

The quotient of the ultimate stress divided by the working stress is called the "*factor of safety*." The Table [Unwin, page 24] gives some general values of the factor of safety for a few of the common materials with constant stress, varying stress of one kind, reversal of stress, and shock. Various writers have given such tables, and a comparison of the factor of safety recommended by different authorities shows a very wide range. See Thurston's Text Book of the Materials of Construction, page 342; Merriman's Mechanics of Materials, page 18, and many others. All such general values should be looked upon simply as suggestions; for the proper factor of safety can only be determined by careful study of the conditions of the particular case in hand. It is frequently proper to use different factors of safety for different members of the same structure or machine. Different materials and the methods of working these materials make some parts more liable than others to hidden defects. Certain members may be subject to considerable variation, or even to reversal of stress, or to shock; while other members carry a load which varies much less. A later article will treat more fully of

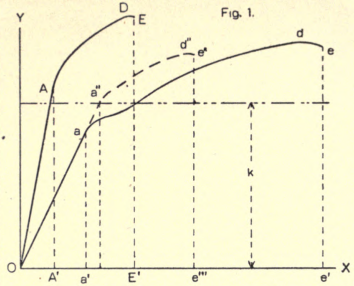

Fig. 1.

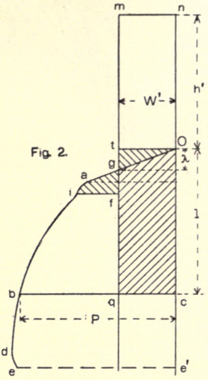

Fig. 2.

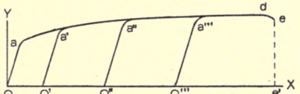

Fig. 3.

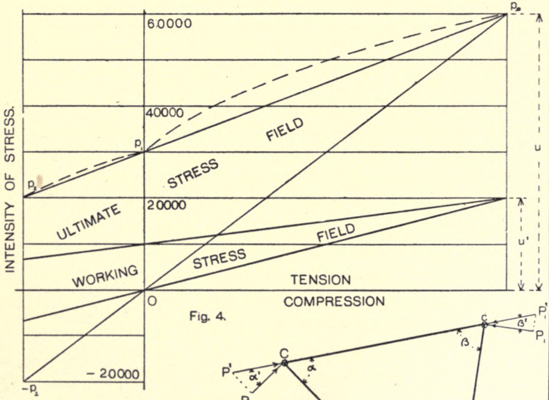

Fig. 4.

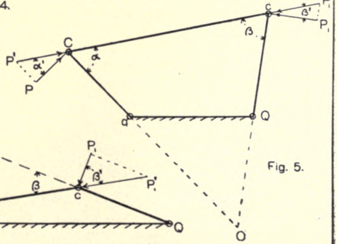

Fig. 5.

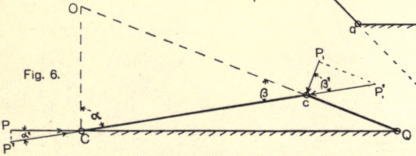

Fig. 6.

the considerations involved in determining the factor of safety appropriate to the cases which ordinarily arise.

5. Steady or Dead Load, and Variable or Live Load.—[Unwin, § 18, pages 24–25.]

6. Stress and Strain.—[Unwin, §§ 18, 19, pages 25–28.]

7. Resilience.—[Unwin, § 23, page 38.] If a material is distorted by a straining action, it is capable of doing a certain amount of work as it recovers its original form. If the deformation does not exceed the elastic strain, this amount of work is equal to the work done upon the material in producing such deformation. If the material is strained beyond the elastic limit, it only returns work equal to that expended in producing elastic deformation; and the energy required to cause the plastic deformation, or set, is not recovered, as it is not stored but has been expended in producing such permanent change of form. Ordinary springs illustrates the first case; the shaping of ductile metals by forging, rolling, wire-drawing, etc., are processes in which nearly all of the energy is expended in producing set.

The work required to produce a strain in a member is called the resilience. If the strain produced is equal to the deformation at the true elastic limit, the energy expended is called *elastic resilience*.* If the piece is ruptured, the energy expended in breaking it is called *ultimate resilience*. If $O\,a\,d\,e$ (Fig. 1) is the stress-strain diagram for a given material; the area $O\,a\,a'$ represents the elastic resilience; and $O\,a\,d\,e\,e'$ represents the ultimate resilience.

In such materials as have well marked elastic limits (proportionality between stress and strain through a definite range) the line Oa is a sensibly straight line, and the elastic resilience, $Oaa' = \frac{1}{2}aa' \times Oa'$; *or, the elastic resilience equals the elastic strain* (Oa') *multiplied by one-half the elastic stress* ($\frac{1}{2}aa'$). The area $Oadee'$ equals the base (Oe') multiplied by the mean ordinate (y) of the curve $Oade$; or, if the quotient of this mean ordinate of the curve divided by the maximum ordinate be called k, the

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* When the term resilence is used without qualifying context, elastic resilence is to be understood.

ultimate resilience equals the ultimate strain multiplied by k times the maximum stress. It is evident that for a straining action beyond the elastic limit, $k > \frac{1}{2}$ and $k < 1$.

The curve $OADEE'$ represents the stress-strain diagram of a material having higher elastic and ultimate strength than the former. The greater inclination of the elastic line (OA) with the axis of strain (OX) shows, in the second case, a higher modulus of elasticity, as this modulus equals the elastic stress divided by the elastic strain. In the first case, $E_1 = \frac{aa'}{Oa'}$; in the second case, $E_2 = \frac{AA'}{OA'}$.

The stress-strain diagram $OADEE'$ shows that of two materials one may have both the higher elastic and ultimate strength, and still have less elastic and less ultimate resilience If the curve $O\,a''\,d''\,e''$ is the stress-strain diagram of a third material, (having a similar modulus of elasticity to the first) it appears that this third material possesses greater elastic resilience, but less ultimate resilience than the first.

A comparison of these illustrative stress-strain diagrams (for quite different materials), also shows that, *for a given stress*, the more ductile, less rigid material has the greater resilience. Hence, when a member must absorb considerable energy, as in case of severe shock, a comparatively weak yielding material may be safer than a stronger, stiffer material. This is frequently recognized in drawing specifications The principle is the same as that involved in the use of springs to avoid undue stress from shock. In fact springs differ from the so-called rigid members only in the *degree* of distortions under loads, or in having much greater resilience for a given maximum load.

If a material is strained beyond its elastic limit, as to a' (Fig. 2), upon removal of the load it will be found to have such a permanent set as $O\,O'$. Upon again applying load, its elastic curve will be $O'\,a'$; but beyond the point a' its stress-strain diagram will fall in with the curve which would have been produced by continuing the first test (*i. e.*, $a'\,d\,e$). Similarly, if loaded to a'', the permanent set is $O\,O''$, and upon again applying load, the

stress-strain diagram becomes $O' a'' d e$. The elastic limit a'' of the overstrained material is evidently higher than the original elastic limit, a; while the original total resilience, $O a d e$, is considerably greater than the total resilience of the overstrained material, $O'' a'' d e$. The effects of strain beyond the elastic limit are thus seen to be:

I. Elevation of the elastic strength and increase of the elastic resilience.

II. Reduction of the total resilience.

These facts have an important influence on resistance to repeated shock. The above noted elevation of the elastic limit by overstraining can usually be largely or wholly removed by annealing.

8. **Suddenly applied Load, Impact, Shock.**—[Unwin, § 18, pages 24–25; § 21a, page 35.] It will perhaps be well to first consider the general case of a load impinging on the member, with an initial velocity; this velocity (v) corresponding to a free fall through the height h. For simplicity, the discussion will be confined to a load producing a tensile stress; but the formulas will apply equally well to compressive and uniform shearing stresses, and all except (3) and (7) apply directly to cases of torsion and flexure.

$W =$ static value of load applied to member.

$h =$ height corresponding to velocity with which load is applied.

$l =$ total distortion of member due to impulsive load.

$p =$ maximum *intensity* of resulting stress.

$A =$ area of cross-section of the member.

$P = p\,A =$ total max. stress due to load as applied.

$\lambda =$ total distortion of member due to static load, W.

$x = h \div \lambda$.

$k =$ a constant; its value is $\frac{1}{2}$ if $E.\ L.$ is not passed; but if $E.\ L.$ is exceeded $k > \frac{1}{2}$ and $k < 1$.

The energy to be absorbed by the member due to the impulsive application of the load is $W(h + l)$; the resilience is $k\,Pl$. (See

preceding art., Resilience.) The expression $W(h+l)$ gives the potential energy corresponding to the kinetic energy, $\frac{W}{2g}v^2$, as given by Unwin on § 23.

Case 1.—Maximum Stress within Elastic Limit.

$$W(h+l) = kPl = \tfrac{1}{2}Pl. \qquad (1)$$

$$\therefore P = \frac{2W(h+l)}{l} \qquad (2)$$

$$p = \frac{P}{A} = \frac{2W(h+l)}{Al}. \qquad (3)$$

$$l : \lambda :: P : W \quad \therefore l = \frac{P\lambda}{W}. \qquad (4)$$

$$P = \frac{2Wh}{l} + 2W = \frac{2Wh}{\frac{P\lambda}{W}} + 2W = \frac{2W^2h}{P\lambda} + 2W. \qquad (5)$$

$$\therefore P^2 = \frac{2W^2h}{\lambda} + 2WP \quad \therefore P = W\left(1 + \sqrt{1 + \frac{2h}{\lambda}}\right)$$
$$= W(1 + \sqrt{1+2x}). \qquad (6)$$

$$p = \frac{P}{A} = \frac{W}{A}\left(1 + \sqrt{1 + \frac{2h}{\lambda}}\right) = \frac{W}{A}(1 + \sqrt{1+2x}). \qquad (7)$$

$$l = \frac{P\lambda}{W} = \lambda\left(1 + \sqrt{1 + \frac{2h}{\lambda}}\right) = \lambda(1 + \sqrt{1+2x}). \qquad (8)$$

If $x = 0$, $h = 0$; $P = 2W$; $l = 2\lambda$.
" $x = 1$, $h = \lambda$; $P = 2.73W$; $l = 2.73\lambda$.
" $x = 4$, $h = 4\lambda$; $P = 4W$; $l = 4\lambda$.
" $x = 12$, $h = 12\lambda$; $P = 6W$; $l = 6\lambda$.
" $x = 24$, $h = 24\lambda$; $P = 8W$; $l = 8\lambda$.
" $x = 40$, $h = 40\lambda$; $P = 10W$; $l = 10\lambda$.

As λ is small for metals (except in the forms of springs) a moderate impinging velocity may produce very severe stress. It will

be evident that λ and l are directly proportional to the length of the member; hence the stress produced by a given velocity of impact (height h) is reduced by using as long a member as possible.

If the load is applied instantaneously, but without initial velocity, $h = 0$ and $x = 0$; and we have:

$$P = W(1 + \sqrt{1 + 0}) = 2W. \quad . \quad . \quad . \quad (6')$$

$$p = \frac{P}{A} = \frac{2W}{A}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7')$$

$$l = \lambda(1 + \sqrt{1 + 0}) = 2\lambda. \quad . \quad . \quad . \quad . \quad . \quad (8')$$

Case II.—Maximum Stress beyond the Elastic Limit. If the maximum stress exceeds the elastic limit, the constant k of eq. (1) is between ½ and 1, (See Art. 7, Resilience), and its exact value cannot be determined in the absence of the stress-strain diagram for the particular material. Thus, (Fig. 3) $W(h + l)$, is represented by the rectangle $mncq$; and this area must equal the resilience area $Oahc$; the latter being greater than the elastic resilience, Oaa', and less than the total resilience, $Oadee'$, in this illustration.

When the stress-strain diagram is known, the following problems can be readily solved:—

(a) Determination of the velocity of impinging of a given load (or corresponding value of h) to produce a given stress, or strain.

(b) Determination of the load which will produce any particular stress, or strain, when impinging with a given velocity.

(c) Determination of the stress, or strain, produced by a given load impinging with a given velocity.

Let the resilience corresponding to the known stress, or strain, in (a) and (b), be called $R = kPl$. If the stress-strain diagram is for stress per unit of sectional area and strain per unit of length of the member, let W' be the load per unit of sectional area, and h' be the height due the velocity of impinging divided by the total acting length of the member.

(a): $W'(h'+l) = kPl = R.$

$$\therefore h' + l = \frac{R}{W'}, \therefore h' = \frac{R}{W'} - l. \quad . \quad . \quad . \quad (9)$$

(b): $$W' = \frac{R}{h'+l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

(c): The solution of this problem is not quite so definite, in the general case, as the preceding; but it can be easily accomplished, graphically, with sufficient accuracy. Draw the line $g\,q$ (Fig. 3) (indefinitely), parallel to $O\,e'$, and at a distance from it equal to W'; take out the area $f\,i\,g = g\,O\,t$. Whatever the value of l, the shaded area $O\,c\,q\,f\,i\,g\,O = W'\,l$; hence the unshaded area under the stress-strain curve must equal $W'\,h'$. A few trials will suffice to locate the limiting line $b\,q\,c$ which will give $f\,i\,b\,q\,f = m\,n\,O\,t = W'\,h'$.

The case in which the maximum stress is within the elastic limit is by far the most important, as it is almost always desired to keep the maximum intensity of stress, $P \div A$, within the elastic limit; especially as every overstrain (beyond this limit) raises the elastic limit and decreases the total resilience [see Fig. 2]. The effect of a shock which strains a member beyond the elastic limit is to reduce its margin of safety for subsequent similar loads, because of reduction in its ultimate resilience. Numerous successive reductions of the total resilience by such actions may finally cause the member to break under a load which it has often previously sustained.

No doubt many cases of failure can be accounted for by the effects just discussed; but there is another and quite different kind of deterioration of material, which is treated in the following article.

Dr. Thurston has shown that the prolonged application of a dead load may produce rupture, in time, with an intensity of stress considerably below the ordinary static ultimate strength but above the elastic stress. It is well known that an appreciable time is necessary for a ductile metal to flow, as it does flow when its section is changed under stress; hence, a test piece will show greater apparent strength by quickly applying the load than by

applying it more slowly, provided the application of load is not so rapid as to become impulsive.

The kind of failure which is the subject of the next topic is due to a real permanent deterioration of the metal, and it is due to distinctly different causes than those mentioned above.

9. On the Peculiar Action of Live Load. Fatigue of Metals. —[Unwin, §21, pages 29–34.] Exception may be taken to the statement (fifth line from bottom of page 29): "Here the factor 2 or 3 is a real factor of safety which allows for unknown contingencies." A factor of *nearly 2* is required to allow for the *known* difference between the ultimate and the elastic strengths; so that when the working stress is ½ the ultimate stress, there is scarcely any margin for contingencies; and with a working stress of ⅓ the ultimate stress the factor for contingencies, above the point at which permanent set is to be expected, is only about $3 \div 2 = 1\frac{1}{2}$.

It has been found by experience and experiment, that materials which are subjected to continuous variation of load cannot be depended upon to resist as great stress as they will carry if applied but once, or only a few times. When the load is suddenly applied, and frequently repeated, the decline of strength or of the power of endurance, may perhaps be ascribed, in part at least, to the elevation of the elastic limit and reduction of the ultimate resilience, as discussed in Art. 8. But apart from this cause, with repeated loads, even in the absence of appreciable shock, a decided deterioration of the material very frequently occurs. This effect has been called the *Fatigue of Materials*, although Unwin (§ 21a) restricts this term to the kind of deterioration already referred to as the simple result of a decrease of resilience. The term fatigue implies a weakening of the material due to a general change of structure. It was formerly commonly supposed that the repeated variation of stress caused such change of structure, possibly owing to slight departure from perfect elasticity under stress much below that ordinarily designated as the elastic limit. The crystaline appearance of the fracture sustained this view; but numerous tests of pieces from a member ruptured in this way, (taken as near as possible to the break), fail to show

such crystaline fracture, and it is difficult to reconcile the normal appearance and behavior of such test pieces with the theory of general change of structure.

Every piece of metal contains innumerable minute flaws or imperfections, often originally too small to be detected by ordinary means. These "micro-flaws" tend to extend across the section under variation of stress, and may, in time, reduce the net sound section so greatly that the intensity of stress in the fibres which remain intact becomes equal to the normal breaking strength of the material. Professor Johnson suggests: "the gradual fracture of metals" as a more appropriate term than "fatigue." Many men of large practical experience still prefer wrought iron to mild steel for various members which are subject to constantly reversing stress.

It is probable that the prejudice against steel is largely the result of unskillful manipulation of this more sensitive material; and the product of the best steel makers of today is much stronger and more reliable than wrought iron.

However, the very lack of homogeneity in wrought iron renders it safer under varying stress, (*other things being equal*), as the fibres are more or less separated by the streaks of slag, and a flaw is less apt to extend across the entire section than it is in the continuous structure of steel. Wrought iron may be likened to a wire rope, in which a fracture in one wire does not directly extend to adjacent wires.

The "gradual fracture" through extension of "micro flaws" seems to accord with the observed facts more closely than the older theory of general change of structure.

The theory of the subject is, as yet, too incomplete to permit of derivation of rational formulas to account for the effects of repeated live loads; and if the "micro-flaw" theory is correct, it is not probable that such rational analysis can ever be satisfactorily applied.

All of the formulas that have been derived for computation of breaking strength under known variations of load, or stress, are empirical ones which have been adjusted to fit the experimentally determined facts.

Consult : Johnson's Materials of Construction.
Merriman's Mechanics of Materials.
Unwin's Testing of Materials.
Weyrauch (Du Bois) Structure of Iron and Steel.

Experiment has shown that the breaking strength under repeated loading, or the "carrying strength", is a function of the magnitude of the variation of stress and of the number of repetitions of such varying stress. Furthermore, this function is different for different materials ; and there are authentic observations on record which go to show that, as between different materials, the one with the higher static breaking strength does not always possess the greater endurance under repeated loading. In general, however, the carrying strength under repeated loads is a function of the static strength.

The allowable working stress usually depends upon : (a), The number of applications. This should be considered as indefinite, or practically infinite, in many machine members. (b), The range of load. This is frequently either from zero to a maximum ; or between equal plus and minus values. (c), The static breaking strength or the elastic strength.

The first systematic experiments upon the effect of repeated loading were conducted by Wöhler [1859 to 1870]. He found, for example, that a bar of wrought iron, subjected to tensile stress varying from zero to the maximum, was ruptured by :

800	repetitions	from	0 to 52,800	lbs. per sq	in.
107,000	"	"	0 " 48,000	"	"
450,000	"	"	0 " 39,000	"	"
10,140,000	"	"	0 " 35,000	"	"

[Merriman, page 191].

It was found that the stress could be varied from zero up to something less than the elastic limit an indefinite number of times (several millions) before rupture occurred ; but with complete reversal of stress, or alternate equal and opposite stresses, (tension and compression), it could be broken, by a sufficient number of applications, when the maximum stress was only about one-half to two-thirds the stress due to the elastic limit.

The general formula given by Unwin, (eq. (1), page 32), for the maximum carrying strength, is:

$$k_{\max} = \frac{\Delta}{2} + \sqrt{K^2 - n\Delta K}$$

This expression can be used when the range of intensity of stress, Δ, (*i.e.*, the *difference* between minimum and maximum unit stress) is known. It is not directly applicable, however, to the general problem of design, which is to determine the *dimensions* of a member to safely carry a *load* varying between given limits; for the variation of unit stress (intensity of stress) due to the range of load is not known until these dimensions are known. Thus, if the load on a wrought iron tension member varies between 3 tons and 20 tons an indefinite number of times, it is desired to determine the proper area of cross-section; but the value of Δ cannot be assigned until this section is known. Three special forms of the above expression are given (Unwin, page 32) for the special cases of: (1) *dead load*, $k_{\max} = k_{\min}$, $\Delta = 0$; (2) *repeated load* (stress entirely removed but never changing sign), $k_{\min} = 0$, $\Delta = k_{\max}$; and (3) *complete reversal of load* (equal and opposite stresses alternating), $k_{\min} = -k_{\max}$, $\Delta = 2k_{\max}$. These special formulas are generally applicable to the appropriate cases, because Δ has been eliminated. However, it appears best to adopt a formula given by Professor Johnson, as it is applicable to all cases that will arise; it is simpler than most of those previously proposed; and it is probably as reliable as any yet offered.

Two formulas which have been very generally accepted for computing the probable carrying strength are: Launhardt's for varying stress of one kind only, and Weyrauch's for stress which changes sign.

Suppose a material to have a static ultimate strength of 60,000 lbs. per sq. in. If the minimum unit strength be plotted as a straight line, $-p_2\,Of$, (Fig. 4), the locus of the maximum unit stress, from the Launhardt formula, is the broken curve from d to f. That is, for example, when the minimum tensile stress is 15,000, the maximum tensile carrying stress would be about

40,000; or the material could be expected to stand an indefinite number of loadings if the range of stress did not exceed 15,000 to 40,000 pounds per square inch tension. In a similar way, the broken curve from c to d is the locus of maximum tension, from the Weyrauch formula, when the locus of minimum stress (negative tension, or compression) is the straight line $-p_2\,O$. It will appear that the straight line $p_2 p_1 f$ agrees fairly well with these two curves. Inasmuch as it seems unreasonable to expect an abrupt change of law when the minimum stress passes through zero, and as there is no rational basis for the Launhardt and Weyrauch formulas, it appears reasonable to adopt the upper straight line as the locus of the maximum stress. Owing to the discrepancies in the observations (which must be expected from the probable cause of the deterioration of the metal), this straight line may be accepted as representing the law as accurately as could be expected of any empirical line. These are, in substance, the reasons given by Professor Johnson for basing his formula on the straight line $p_2 p_1 f$. For full discussion and derivation of the following formula, see Johnson's Materials of Construction, pages 545–547.

Let $p =$ maximum intensity of stress.
$p' =$ minimum intensity of stress.
$u =$ ultimate (static) intensity of stress.
$u' =$ ultimate (static) intensity of stress divided by the proper factor of safety.
$f =$ working stress.

Then:

$$p = \frac{\frac{1}{2}u}{1 - \frac{1}{2}\frac{p'}{p}}. \qquad (1)$$

$$f = \frac{\frac{1}{2}u'}{1 - \frac{1}{2}\frac{p'}{p}}. \qquad (2)$$

As the expressions contain the *ratio* of the minimum to maximum intensities of stress, instead of their *difference*, they are applicable when the area of cross-section of the member is un-

known; for whatever this area, the ratio of the stresses is the same as the ratio of the loads producing these stresses. In substituting values of p and p', care must be taken to use proper signs; thus, if tension is taken as positive, compression is negative; or, if the stress varies between tension and compression, p is positive and p' is negative.

For dead load, $p' = p$;

$$\therefore p = \frac{\frac{1}{2}u}{1 - \frac{1}{2}\frac{p}{p}} = \frac{\frac{1}{2}u}{\frac{1}{2}} = u. \qquad (1')$$

$$\therefore f = \frac{\frac{1}{2}u'}{1 - \frac{1}{2}\frac{p}{p}} = \frac{\frac{1}{2}u'}{\frac{1}{2}} = u'. \qquad (2')$$

For repeated load when $p' = 0$, or $\frac{p'}{p} = 0$

$$\therefore p = \frac{\frac{1}{2}u}{1 - 0} = \frac{1}{2}u. \qquad (1'')$$

$$\therefore f = \frac{\frac{1}{2}u'}{1 - 0} = \frac{1}{2}u'. \qquad (2'')$$

For complete reversal of load, $p' = -p$,

$$\therefore p = \frac{\frac{1}{2}u}{1 - \frac{1}{2}\frac{-p}{+p}} = \frac{\frac{1}{2}u}{1 + \frac{1}{2}} = \frac{1}{3}u. \qquad (1''')$$

$$\therefore f = \frac{\frac{1}{2}u'}{1 - \frac{1}{2}\frac{-p}{+p}} = \frac{\frac{1}{2}u'}{1 + \frac{1}{2}} = \frac{1}{3}u'. \qquad (2''')$$

10. The Real Factor of Safety.—[Unwin, page 34. *Determination of Safe Working Stress.*] As shown in the preceding article, the safe working stress for a given material, when subjected to repeated variation of load, should be less than that which could be safely allowed for the same material under a dead load.

Machine members are usually subjected to varying stresses; the load is frequently applied so rapidly as to constitute an impulsive action; and the kinetic conditions often introduce considerable additional stress of such complex nature as to preclude very exact analysis. Owing to these elements, a lower working stress is necessary in many machine members than is required, for equal safety with the same material, in stationary structures subjected to a nearly constant load.

The *apparent factor of safety*, which is the quotient of the static ultimate strength divided by the working stress, is often 8, 10, 12, or more, in machine parts; while it may be only 3 or 4 in a structure with a nearly steady load. But it does not necessarily follow that the *real factor of safety*, or margin allowed for such contingencies as defects of material and workmanship is any larger in the former than in the latter case.

In fact the usual margin for such contingencies is not larger in machine members, or there would be correspondingly fewer failures of these parts in regular service. It is true that the elements of uncertainty in the total straining action on a machine are often more numerous, and of greater magnitude relative to the primary straining action due to the "useful load"; hence the total contingency factor may properly be greater than in a bridge or roof truss.

The factor of safety has been called the "factor of ignorance," and as it is too often applied it is perhaps little else. It is probable that the factor of safety must always retain an element of ignorance; for it can hardly be hoped that the powers of analysis will ever permit the prediction of the exact effect of every possible straining action, due to regular service and accident; neither can it be expected that the methods of manufacture and of inspection will become so perfect as to eliminate, or measure precisely, every possible defect in materials and workmanship.

The development of the theory of actions which are as yet but imperfectly understood, may reduce the element of ignorance. However, a careful study of the conditions of each particular case, and proper attention to effects which may be weighed (at least approximately) in the present state of knowledge should

lead to a much more intelligent employment of the factor of safety than is common today.

It has been said that: "The capacity to decide upon the proper factor of safety is *the* important point in design." It is certainly not reasonable to make long, tedious computations, the results of which depend upon a carelessly chosen factor of safety. One who cannot determine a rational factor of safety, can derive little benefit from the use of a rational formula. In short, the selection of the proper constants is the part of the engineer; the computation is the part of a clerk.

Most of the formulas of mechanics, as applied to questions of strength in design, are based upon theoretical treatment of the stresses induced by the action of given forces on the parts under consideration. There are many cases in which this course is perfectly logical, and the conclusions are irresistible; while, in many other instances, members of a machine or structure are subjected to such a complicated system of stresses that analysis cannot be strictly applied, and less satisfactory approximations, or assumptions, are unavoidable, in the present state of knowledge. This last condition of things, which is not unusual in the design of machines, introduces the first of many elements of uncertainty, and one of three methods of arriving at the proportions of the parts is possible. *First*, if the predominating action is capable of rational treatment, the member can be designed as if for the corresponding stresses, and such a margin as is dictated by experience or experiment may then be allowed for the more uncertain elements. *Second*, analysis may be abandoned and resort may be had to empirical formulas derived from experiment. *Third*, the last, and not most uncommon, recourse is to "judgment." This last method, when it is *real* judgment, based upon a large experience, has produced magnificent results; in many cases, (especially for details and small parts), it is the only way to proceed.

The general nature of the factor of safety, and the effects of shock and of repeated stresses have been discussed in preceding articles.

If the working stress due to the total regular straining action,

and the stress which the material will sustain indefinitely (under the conditions to which it is subjected) are known, it is only necessary to so proportion the members that the latter stress will exceed the former by margin enough to cover such contingencies as over loading and defects of material and of workmanship as might reasonably be expected to possibly occur.

The following comparisons give an idea of what this contingency margin, or *real factor of safety* is, with such working stresses as are not uncommonly allowed in machines. The Static Breaking Strengths of the various materials are assumed as fair representative values. Each Working Stress is obtained by dividing the static breaking strength by the appropriate apparent factor of safety as given in the table on page 24 of Unwin's Machine Design. The Carrying Strength is taken as follows:

Case I. Dead Load ;—Carrying strength = static strength.

Case II. Repeated Load,—applied and removed an indefinite number of times but stress of one kind only (tension) ;—Carrying strength = ½ static strength.

Case III. Reversal of Load,—stress varying an indefinite number of times between equal tension and compression,—Carrying strength = ⅓ static strength.

The *Real Factors of Safety* are obtained by dividing the *Carrying Strengths* by the corresponding working stresses.

		CAST IRON.	WRO'T IRON.	MACH'Y STEEL.
CASE I.	Carrying Strength	17,000	56,000	65,000
	Working Stress	4,250	18,670	21,670
	Real Factor of Safety	4	3	3
CASE II.	Carrying Strength	8,500	28,000	32,500
	Working Stress	2,830	11,200	13,000
	Real Factor of Safety	3	2½	2½
CASE III.	Carrying Strength	5,670	18,670	21,670
	Working Stress	1,700	7,000	8,125
	Real Factor of Safety	3⅓	2⅔	2⅔

It will appear from the above table that the real factors of safety are rather less for Cases II and III than those taken for a dead load (Case I); hence the high apparent factors do not provide an excessive margin for the contingencies likely to occur. In fact this margin is comparatively small; for the liability to extra straining action is greater with live load than with dead load.

If there is apt to be much shock, the resulting stresses may be much beyond those due to the gradual application of the load, as shown in article 8; and it will be evident that the apparent factors of safety of 15 and 12, given by Unwin for cast iron and wrought iron or steel, respectively, are not excessive under such conditions. In some instances, shock is so violent and indeterminate that the necessary factor of safety becomes so large as to render computations of very little value in proportioning members, and the practical machines of this class are the products of a process of evolution.

The importance of a knowledge, upon the part of the designer, of the methods employed in the manufacture of the materials used, and of the practice of the shops in which the designs are executed, is appreciated when it is considered that these things all have a direct influence upon the proper factor of safety. As improved methods of the metallurgist insure a more reliable and homogeneous product, and as methods of inspection are perfected, the danger from hidden defects in the material furnished becomes less; as artisans become more accustomed to the properties of the material which they handle, and learn to respect its weaknesses; as investigators develop the effect of repeated stresses of the various kinds; and as the engineer learns to study all of these elements and to give to each its due weight; the factor of safety will be reduced, it will become less and less a factor of ignorance, and more and more a true factor of safety

11. Straining Action due to Power transmitted.—[Unwin, § 22, page 36.]

The expression, $P = \frac{550}{v} HP$, may be used to find the strain-

ing force in a chain transmitting power between sprocket wheels; provided, (as is usual), the chain is so loose that only the driving side is under any considerable tension. With endless belt or rope transmission, where the friction between the band and the wheels is depended upon to prevent undue slipping, it is necessary to have considerable tension on the "slack" side to secure sufficient adhesion. In such cases, the above expression gives the *effective* pull due to the power transmitted, or the *difference* between the total tensions on the two straight portions of the band. The maximum straining force on the tight, or driving, side is equal to P (as given above) plus the pull on the slack side; this total tension on the driving side is frequently twice P, or even more.

It is often convenient to use the velocity in feet per minute (V), in place of the velocity in feet per second (v) of the transmitting connector (link, rod or band). $V = 60\,v$,

$$\therefore\ P = \frac{550}{v}\,H\,P = \frac{33000}{V}\,H\,P. \qquad (1)$$

Whatever the path of the point of application of the load, the resultant force acting upon this point (due to the load combined with the constraining forces) must lie along the tangent to the path. [Newton's Laws.]

When the path of the moving connector does not coincide with the line of its axis, the total straining force along this axis can be found by multiplying P (computed as above) by the secant of the angle which the direction of the load force makes with the direction of the axis of the connector.

In Figs. 5 and 6, let:—

P = useful load force applied at C,
P_1 = corresponding resistance overcome at c,
P' = resultant force at C, along axis $C\,.\,.\,c$,
P_1' = resultant force at c, along axis $C\,.\,.\,c$,
P'' = constraining force at C,
P_1'' = constraining force at c.
$\alpha' = 90° - \alpha$; $\beta' = 90° - \beta$.

$$P' = P \sec \alpha' = P \operatorname{cosec} \alpha = P \div \sin \alpha. \qquad (2)$$

$$P_1' = P_1 \sec \beta' = P_1 \operatorname{cosec} \beta = P_1 \div \sin \beta. \qquad (3)$$

It is evident that P_1' is equal and opposite to P'; or $P_1' = -P'$, if the connector (Cc) is moving with uniform velocity; for considering the connector as a free body acted upon by these two opposite resultant forces, inequality of these forces would produce acceleration (positive or negative).

The resistance (P_1) overcome at c is only equal to the driving force (P) acting at C, when these two points have equal velocities; for the energy applied at C equals the energy delivered at c (neglecting losses due to friction); hence the forces acting at C and c are inversely as the velocities of these points. The resistance (P_1) corresponding to a known driving force (P) can be found as follows:

From the necessary conditions for equilibrium, Σ moms $= 0$:

$$P \cdot OC = P_1 \cdot Oc$$

$$OC : Oc :: \sin \beta : \sin \alpha$$

$$\therefore P \sin \beta = P_1 \sin \alpha. \tag{4}$$

The expression $M = \dfrac{550\, HP}{2\pi n}$ corresponds with $P = \dfrac{550\, HP}{v}$;

for if $r =$ the radius at which the force P acts, $v = 2\pi r n$

$$\therefore P = \frac{550\, HP}{2\pi r n}, \therefore Pr = M = \frac{550\, HP}{2\pi n}. \tag{5}$$

If r is in inches and P in pounds, M is the moment in inch-pounds.

The expression: $M = 63024 \dfrac{HP}{N}$ should be committed to memory.

12. Straining Actions due to Variations of Velocity.—[Unwin, § 22, pages 36–37.]

If the acceleration, $\dfrac{dv}{dt}$, be represented by $\pm p$, the force required to produce this acceleration is $\pm \dfrac{W}{g}\dfrac{dv}{dt} = \pm \dfrac{W}{g} p$, and the stress produced in a member which transmits this force is $\mp \dfrac{W}{g} p$.

Unwin refers to p (page 37) as the acceleration "per unit of weight", and he calls $\frac{W}{g}p$ the "total acceleration." It is more exact to consider p as the acceleration (rate of change of velocity), which is numerically equal to the *accelerating force per unit of mass*, and $\frac{W}{g}p$ as the *total accelerating force.*

Referring to Unwin's Fig. 6, let the angle which the tangent at b makes with the axis, Ox, be called α. Then the velocity ($v, = ab$) is increasing at a rate which is proportional to $\sin \alpha$; and the distance ($s, = Oa$), or space passed over, is increasing at a rate proportional to $\cos \alpha$. If the unit of velocity and unit of space passed over are plotted to the same scale, ad represents the acceleration to a similar scale, for :

$$dv : ds :: \sin \alpha : \cos \alpha \text{; also,}$$

$$ad : ab :: \sin \alpha : \cos \alpha,$$

$$\therefore dv : ds :: ad : ab$$

$$\therefore \frac{dv}{dt} : \frac{ds}{dt} :: ad : ab$$

But, $\frac{dv}{dt}$ = the acceleration, and

$\frac{ds}{dt}$ = the velocity ; therefore if ab = the velocity to any scale, ad = the acceleration to a corresponding scale.

The accelerating force (F) equals the mass multiplied by the acceleration, or $F = \frac{W}{g}p = \frac{W}{g} \times ad$. In a velocity diagram on a *space* base, (*i. e.*, with abscissas representing space traversed, and ordinates representing corresponding velocities) the subnormal (ad) gives the acceleration. In velocity diagrams plotted on a *time* base (*i. e.*, abscissas representing time instead of distance) the subnormal does not give the acceleration.

13. Stress due to Change of Temperature. [Unwin, § 23, page 38].

If a bar is subjected to a change of temperature, of t degrees, it would, if free to expand, increase in length from l to $l(1 + \alpha t)$; α being the coefficient of expansion. If expansion is prevented by a constraining force, this force equals that required to shorten the bar by the amount $l \alpha t$, or to shorten each unit of length by αt. Hence the intensity of stress, f, due to prevention of expansion is that corresponding to the unit strain αt Since the modulus of elasticity, E, equals the stress divided by the corresponding strain (within the elastic limit), $E = \frac{f}{a t} \therefore f = E a t$.

The following values may be taken for the coefficient of expansion per one degree Fahrenheit,

Cast Iron	=	.0000062.
Wrought Iron	=	.0000068.
Soft Steel	=	.0000060.
Hardened Steel	=	.0000070.
Brass	=	.0000105.
Bronze	=	.0000106.

II.

COMPOUND STRESS.

14. Resistance of Columns, or Long Struts.—[Unwin, §§ 37–39, pages 77–81 ; omitting Grashof formulas.]

Very short compression members, of ductile material, fail under stress corresponding to, or only slightly in excess of, the apparent elastic limit, or yield point ; for when this stress is reached the metal flows, although it does not actually break. Very long columns may approximate the resistance as given by Euler's formulas Columns of lengths intermediate between compression members which yield by simple crushing and those which fail by pure flexure are weaker than the former and stronger than the latter. If a column is initially exactly straight, perfectly homogeneous, and subjected to an absolutely concentric load (that is, if it is an ideal column) there seems to be no reason why its strength should diminish rapidly with an increase of length, other conditions remaining the same.

However, even an ideal very long column would reach the condition of unstable equilibrium when subjected to a certain critical load (the greatest load consistent with stability). If the load is increased beyond this limit and a deflection is caused in any way, the deflection will increase until the stress due to flexure produces failure of the column. If a deflection is caused while the column is under a load less than this greatest load consistent with stability, the elasticity of the material tends to make the column regain its normal form. Initial defects in the form or structure of a column or eccentric application of load tend to produce such a deflection ; hence long struts fail under smaller loads than short struts of similar material and cross-section, for ideal conditions are not realized in practice ; or for equal safety under

a given load long columns must have a greater cross-section, and lower mean, or nominal, working stress.* Even in columns of moderate length, if of ductile material, the flow at the yield point causes buckling.

Merriman says that if the length of a compression member be only from four to six times its least "diameter," it may be treated as one which will yield by simple compression. Unwin (page 81) gives limits within which the Euler formula should not be applied. Other authorities give somewhat different limits; but nearly all agree that most of the columns in ordinary structures and machines are intermediate between simple compression members and those to which Euler's formulas apply. The limits assigned by Professor Johnson for "square ended" columns agree substantially with those given for wrought iron by Unwin. There have been a great many column formulas proposed. A graphical representation of several of these formulas is shown in Fig. 7. In this diagram, abscissas represent ratios of the length of column to the least radius of gyration of the cross-section, and the ordinates represent the nominal (mean) intensity of compressive stress. Or,

$x = l \div \rho = l \div \sqrt{I \div A}$, and $y = p = P \div A$, when

l = the length of the column,

ρ = the least radius of gyration of cross-section,

I = the least moment of inertia of cross-section,

A = the area of the cross-section,

P = the *breaking* load on the column,

p = the *mean* intensity of stress under the *breaking* load, or the *unit* breaking load, $= P \div A$.

The following additional notation is also used in this article:

n = the factor of safety,

*Owing to the flexure of the long column, the stress is not uniform across the section. The maximum intensity of stress must be kept within the compressive strength of the material; hence the mean stress is less than for shorter compression members, in which the mean stress is more nearly equal to the maximum.

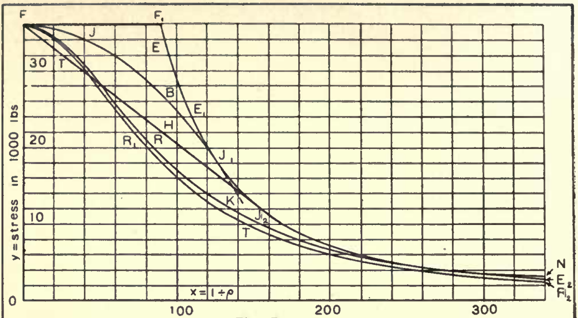


Fig. 7.

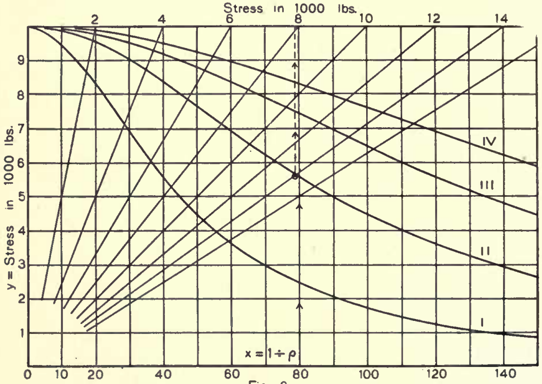


Fig. 8.

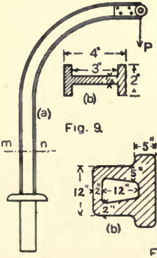


Fig. 9.

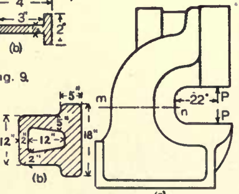


Fig. 10.

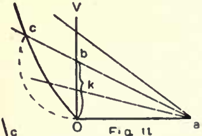


Fig. 11.

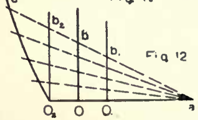


Fig. 12.

$P' =$ the *working* load on the column, $= P \div n$,

$p' =$ the *mean* intensity of *working* stress, or *unit* working load, $= P' \div A$,

$F =$ the crushing strength of the material, or stress at the yield point. This is the maximum intensity of stress in the column when the mean intensity of stress is p.

$f =$ the intensity of working stress in the column $(= F \div n)$. This is the maximum intensity of stress in the column when the mean intensity of stress is p'.

$m =$ a coefficient for the end conditions.

For end conditions as in Table VIII (Unwin, page 80):—

I. *Fixed* at one end and *free* at the other, - - $m = \frac{1}{4}$;
II. "Pin ended" (both ends free but guided), - $m = 1$;
III "Pin and square" (one end fixed the other guided). $m = \frac{9}{4}$;
IV. "Square ended" (both ends fixed), - - - $m = 4$.

The diagram of Fig. 7 is for the ultimate resistance of pin ended columns with a material having a crushing resistance, F, (yield point) of 36,000 pounds per square inch, and a modulus of elasticity, E, of 29,400,000. The value of p is 36,000 for a very short compression member, and it is evident that a long column could not be expected to have a greater strength; hence no formula should be used which would give a value of p in excess of the crushing resistance F. Referring to the diagram, it will appear that the Euler formula (represented by the curve $E\,E_1\,E_2$) cannot apply to columns (of this particular material) in which $l \div \rho < 90$. If columns with a ratio of l to ρ less than this limit yielded by simple crushing, and those with a greater ratio of l to ρ followed Euler's formula, the straight line $F\,F_1$ and the curve $F_1\,E_1\,E_2$ would give the laws for all lengths of columns. It is not reasonable to expect such an abrupt change of law in passing this limit $(l \div \rho = 90)$; and, as already stated, columns of moderate length fail under a mean stress considerably less than the simple crushing resistance of the material; or the strength of columns is inversely as some function of the length divided by the diameter.

Mr. Thomas H. Johnson has developed a formula which is

based on the assumption that the strength of the column may be taken inversely as $l \div \rho$. This expression is

$$p = \frac{P}{A} = F - k\frac{l}{\rho}, \qquad (1)$$

in which the coefficient k has the value,

$$k = \frac{F}{3}\sqrt{\frac{4F}{3m\pi^2 E}}.$$

This formula is represented by the straight line THJ_2 in Fig. 7. It will be noted that this line is tangent to the Euler curve at J_2, and the equation of the latter is to be used, should the columns exceed the length corresponding to this point of tangency, $(l \div \rho > 150)$. This expression is very simple, after k has been determined. It is very convenient in making a large number of computations for columns of any one material, and it is employed in bridge design to a considerable extent. It does not appear to have any advantage, on the ground of simplicity, when some particular value of k does not apply to several computations. Furthermore, this formula gives rather large sections for columns in which $l \div \rho$ is less than about 40.

For determination of nominal working stress, p (as computed above) may be divided by a suitable factor of safety, n. Or if $p \div n = p'$, the expression may be put in the following form for direct computation of mean working stress:

$$p' = \frac{F}{n} - \frac{k}{n}\frac{l}{\rho} = f - \frac{f}{3}\sqrt{\frac{4F}{3m\pi^2 E}}\,\frac{l}{\rho}. \qquad (2)$$

Professor J. B. Johnson has derived a formula from the results of the very careful experiments of Considère and Tetmajer. His formula is:

$$p = F - \frac{F^2}{4\pi^2 E}\left(\frac{l}{\rho}\right)^2 \qquad (3)$$

for pin ended columns. The curve JBJ_1 (Fig. 7) represents this expression. This curve is a parabola tangent to the Euler curve, and with its vertex in the axis or ordinates at F, the

direct crushing stress of the material. For columns having $l \div \rho$ greater than the value corresponding to the point of tangency J_1, (should such be used)), the Euler formula is to be employed. This formula of Professor Johnson's is empirical, but it agrees remarkably well with very refined experiments on *breaking* loads. It gives considerably higher values for allowable stress than other generally accepted formulas, probably because it is based upon more refined tests, or upon conditions further removed from those of practice.

Professor Johnson says (Materials of Construction, pages 301–302) that both Bauschinger and Tetmajer "mounted their columns with cone or knife-edge bearings at the *computed* gravity axis, while M. Considère mounted his with lateral-screw adjustments, and arranged a very delicate electric contact at the side so as to indicate a lateral deflection as small as 0.001 mm. He then applied moderate loads to the columns and adjusted the end bearings until they stood under such loads rigidly vertical, with no lateral movement whatever."*

It would appear that this precaution tends to make the test one of the *material and not of a long strut;* for the eccentricity of the load (relative to the nominal geometric axis) compensates, in a measure, for the lack of homogeniety of the material. Had the correction been made under greater load, the results of the tests, if plotted in Fig. 7, would probable be still nearer the line FF_1, and the difference between these test columns and columns as used in practice would be greater, requiring a higher contingency factor in the latter for safety.

For determining the working stress, the value of p (as computed from the above form of Johnson's expression) should be divided by a suitable factor of safety n. Or, the formula may be put in the following form for computing nominal working stress:

$$p' = f - \frac{Ff}{4\pi^2 E}\left(\frac{l}{\rho}\right)^2. \qquad (4)$$

*"This precaution is essential to a perfect test of the *material* * * * Only in this way can other sources of weakness be eliminated."—[J. B. J.]

The Rankine, or Gordon, formula (see Church's Mechanics, pages 372–376) has been extensively used for columns. It may be expressed as follows:

$$p = \frac{P}{A} = \frac{F}{1 + m\beta\left(\frac{l}{\rho}\right)^2}. \qquad (5)$$

The above formula is based upon experiments on the *breaking* strength of columns. The coefficient β is purely empirical, and this fact limits its usefulness, for it leaves much uncertainty as to how this coeffcient should be modified for different materials than those which have been actually tested as columns. The mean intensity of working stress, p', might be inferred by dividing p by n, or the expression can be written;

$$p' = \frac{f}{1 + \beta\left(\frac{l}{\rho}\right)^2}; \qquad (6)$$

but it is not entirely satisfactory to assume the action for stresses within the elastic limit, from the results of tests for breaking strength. The *form* of the Rankine expression is rational, but the coefficient β is not.

Professor Merriman says, in his Mechanics of Materials, page 129: "Several attempts have been made to establish a formula for columns which shall be theoretically correct . . . The most successful attempt is that of Ritter, who, in 1873, proposed the formula

$$p' = \frac{P'}{A} = \frac{f}{1 + \frac{F}{m\pi^2 E}\left(\frac{l}{\rho}\right)^2}. \qquad (7)$$

"The form of this formula is the same as that of Rankine's formula, . . . but it deserves a special name because it completes the deduction of the latter formula by finding for β a value which is closely correct when the stress f does not exceed the elastic limit F." The above notation is changed to agree with that previously used in this article. The ratio $F \div f$ is the factor of safety. For ultimate strength, this formula might be written:

$$p = \frac{P}{A} = \frac{F}{1 + \frac{F}{m\,\pi^2 E}\left(\frac{l}{\rho}\right)^2}, \qquad (8)$$

but the first form (eq. 7) is the more important. The curve $R_1\,T\,R_2$ (Fig. 7) is the graphical representation of the last expression, eq. 8.*

Merriman gives the Euler formula for a factor of safety of $n = F \div f$, which is

$$p' = \frac{P}{A\,n} = \frac{f}{F}\,m\,\pi^2 E\left(\frac{\rho}{l}\right)^2. \qquad (9)$$

Failure occurs if $f \geqq F$. The Ritter formula (eq. 8) reduces to this last expression for columns so long that the term unity in the denominator is negligible; strictly speaking, this is only the case when $l \div \rho =$ infinity. Professor Merriman also shows, mathematically, that the two curves $E\,E_1\,E_2$ and $R_1\,T\,R_2$ are tangent to each other when $l \div \rho =$ infinity.

If $l \div \rho = 0$, the Ritter formula reduces to $p' = P' \div A$, which is the ordinary formula for short compression members.

The fact that this formula is rational in form, that it gives the correct values at the limits $l \div \rho = \infty$ and $l \div \rho = 0$, and that it lies wholly within the boundary $F\,F_1\,E_1\,E_2$ (Fig. 7) all justify its use, and it will be adopted in this work. It will be noted from Fig. 7 that the Ritter and Rankine formulas agree very closely for the material taken for illustration; but the fact that the curve of the latter crosses the Euler curve near the right hand limit of the diagram indicates that its constant β is not theoretically correct.

All of the above formulas give the value of the mean ultimate stress ($p = P \div A$), or the mean working stress ($p' = P' \div A$), corresponding to a maximum ultimate stress F, or a maximum working stress f, respectively. However, the ordinary problem of design is to assign proper dimensions for the member under

*Professor Merriman developed equation 8, independently, but later than Ritter. He gives Ritter sole credit for the formula in the recent (1897) edition of his Mechanics of Materials.

the given load. It is not practicable to solve directly for the area in such expressions as those given in this article as p' (or p) and ρ are both functions of the area of the cross section. It is usual to assume a section somewhat larger than that demanded for simple crushing, and then to check for the ultimate load P, or the working load P'. Mr. W. N. Barnard has devised a diagram which is very convenient for these computations. It is shown, to a reduced scale, in Fig. 8. The four curves are for the four end conditions given on page 27 (or Unwin, Table VIII, page 80). They are plotted for a maximum working stress of 10,000 pounds per square inch ; but may be used for any other stress by proceeding as follows: Assume a trial cross-section, which fixes ρ. Divide l by this value of ρ ; take this quotient on the lower scale and pass directly upward to the proper curve for the given end conditions ; then pass horizontally to that one of the radiating diagonals which is numbered to correspond with the selected stress ; from this last point pass upward to the horizontal scale at the top of the diagram, where the value of the unit load or mean working stress, (p'), is read off.[1] If this value of p' agrees sufficiently well with the quotient of the load divided by the trial area, the section may be considered as satisfactory.

In the case of a square-ended column, or when the supporting action of the ends is equal in all possible planes of flexure, it is sufficient to take the least radius of gyration of the section ; or to take ρ for the axis about which the section is weakest. In case of a pin-ended column, as a connecting rod, the cylindrical supporting pins make it equivalent to a square-ended column against flexure in the plane of the axes of the pins, provided these bear symmetrically with reference to the axis of the column ; while the column is pin-ended with reference to a plane perpendicular to the axes of the pins. If the cross-section of such a column has equal dimensions in these two planes (circular, square sections, etc.), the column need only be computed for the latter

[1] The method of using the diagram is indicated by the arrows, for an example in which $l \div \rho = 80$ and the maximum working stress $= 14,000$. In this case, p' is found to be about 7,900.

plane. If the pin-ended column has an oblong section (elliptical, rectangular but not square, I section, etc.), it may be weaker in either of these two planes, notwithstanding the difference in end conditions relative to them ; and it may be necessary to compute for both planes, unless the section is obviously stronger in one of them. If a rectangular, or elliptical, column has a section in which the dimension in the plane of the pins is more than one half the dimension in the plane perpendicular to the pins, it will suffice to compute as a pin-ended column against flexure in the latter plane, and *vice versa*. In the preceding discussion, the various formulas have been given both for breaking and for working loads. The Euler and Ritter formulas are derived from the theory of elasticity ; hence these are proper for computations pertaining to working loads, in which the stress should never exceed the elastic limit * It does not follow that these two rational formulas will agree with experiments on the ultimate resistance of columns. These expressions are, in this respect, like the common beam formulas. Such formulas as Rankine's and J. B. Johnson's, derived from tests of ultimate resistance of columns, are, for similar reasons, less rigidly applicable to working loads and stresses.

15. Eccentric Load. Tension, or Compression, Combined with Bending.—[Unwin, § 43, pages 89–90.]

It is not practicable to solve the equation

$$f = \frac{P}{A} + \frac{Pr}{Z} \qquad (1)$$

for the direct determination of the dimensions of cross section to sustain a given eccentric load (P) with an assigned intensity of stress (f), because both A and Z are functions of the required dimensions. With solid square or circular sections, or in general when only one dimension is unknown, it is possible to reduce the above equation to a form which can be solved for this unknown quantity ; but the algebraic expression is a troublesome cubic

*The Euler formula is not applicable for practical applications, except for quite long columns.

equation. The practical method is to assume a trial section and check this for either P or f.

Example 1. A small crane (Fig. 9a) has a clear swing of 28 inches. The section at $m \,.\,.\, n$ is shown by Fig. 9b. Find the load corresponding to a maximum fibre stress (compression) of 9000 pounds per square inch at n.

$$f = \frac{P}{A} + \frac{Pr}{Z} \quad \therefore \quad P = \frac{f\,A\,Z}{Z + r\,A}.$$

$$r = 28 + 2 = 30\,; \quad A = 2 \times 4 - 1.5 \times 3 = 3.5\,;$$

$$Z = \frac{1}{6 \times 4}\left(2 \times 64 - 1.5 \times 27\right) = 3.65 \text{ (See Unwin, p. 58).}$$

$$\therefore \; P = \frac{9000 \times 3.5 \times 3.65}{3.65 + 30 \times 3.5} = 1060 \text{ lbs.}$$

Example 2. A punching machine (Fig. 10a) has a reach of 22″. Maximum force (P) acting at the punch is taken at 70,000 lbs. Design section $m \,.\,.\, n$ so that maximum fibre stress at n (tension) shall be about 2,400 pounds per square inch.

The general form of section best adapted for this case is that shown in Fig. 10b Taking the trial dimensions as in Fig. 10b, the neutral axis is found to be 8″ from n.

$\therefore\; r = 22 + 8 = 30$. It is also found that $A = 216$, and $Z = 960$

$$\therefore\; f = \frac{70000}{216} + \frac{70000 \times 30}{960} = 325 + 2200 = 2525 \text{ lbs.}$$

This is slightly greater than the limit assigned for maximum intensity of working stress. If this excess is not considered permissible, a somewhat stronger section is to be taken, checking the latter if this step seems necessary.

16. Combined Torsion and Flexure.—[Unwin, § 44, pages 90–91.]

The expression (Unwin, eq. 27, page 90),

$$M_e = \tfrac{1}{2} M + \tfrac{1}{2} \sqrt{M^2 + T^2} \qquad (1)$$

may be obtained from the relation

$$f_n = \tfrac{1}{2}[f + \sqrt{f^2 + 4f_s^2}]^* \qquad (2)$$

in which f_n = the maximum intensity of normal stress due to the combined bending and twisting moments; f = the intensity of stress due to the simple bending moment; and f_s = the intensity of shearing stress due to the simple twisting moment.

$$M_e = \frac{f_n I}{r},\ M = \frac{f I}{r},\ \ T = \frac{f_s I_p}{r} = \frac{2 f_s I}{r}$$

[$I_p = 2\ I$, for circles, or other sections in which the moments of inertia about two perpendicular axes are each equal]. Hence,

$$M_e = \frac{I}{r}\ f_n = \tfrac{1}{2}\ \frac{I}{r}\left[f + \sqrt{f^2 + 4f_s^2}\right] =$$

$$\tfrac{1}{2}\frac{I}{r} f + \tfrac{1}{2}\sqrt{\frac{I^2}{r^2}f^2 \times 4\frac{I^2}{r^2}f_s^2} = \tfrac{1}{2}M + \tfrac{1}{2}\sqrt{M^2 + T^2} \qquad (3)$$

Since $M = \dfrac{f I}{r}$ and $T = \dfrac{2 f_s I}{r}$, for an *equal intensity of stress* $(f = f_s)$ in bending or twisting, $T = 2\ M$, for the same section. It is, therefore, allowable to substitute for M_e (the bending moment equivalent to the combined bending and twisting moments) $\frac{1}{2}\ T_e$ (the twisting moment equivalent to the combined bending and twisting moments). Then

$$\tfrac{1}{2}\ T_e = \tfrac{1}{2}M + \tfrac{1}{2}\sqrt{M^2 + T^2},$$

$$\therefore\ \ T_e = M + \sqrt{M^2 + T^2}. \qquad (4)$$

In a similar way, eq. (27a) of Unwin, may be reduced to

$$T_e = 2\ M_e = \tfrac{3}{4}\ M + \tfrac{5}{4}\sqrt{M^2 + T^2} \qquad (5)$$

The expressions designated above as equations (3) and (4), are two forms of Rankine's formula for combined bending and twisting, while equation (27a) [Unwin] and the above derived form eq. (5), are due to Grashof. Some authorities use the latter, but the Rankine formula is adopted by Unwin, and by many others, and it will be followed in the present work.

* See Church's Mechanics, eq. (8), page 317, in which $q\ max = f_n$, $p = f$, and $p_s = f_s$, as in eq. (2) above.

If the ratio of M to T be called k, $M = k\,T$; hence eq. (4) reduces to

$$T_e = [k + \sqrt{k^2 + 1}]\,T \qquad (6)$$

and eq. (5) reduces to

$$T_e = [\tfrac{3}{4}\,k + \tfrac{5}{4}\sqrt{k^2 + 1}]\,T \qquad (7)$$

Convenient graphical solutions of equations (6) and (7) are shown in Fig. 11 and 12, respectively.

For solution of the Rankine formula, eq. (6), make $O\,a$ = unity (Fig. 11); lay off $O\,b = k$, to the same scale on the vertical axis $O\,V$; draw $a\,b$, extending it beyond b for a length somewhat greater than k; then, $a\,b = \sqrt{O\,b^2 + O\,a^2} = \sqrt{k^2 + 1}$. Next, lay off $b\,c = O\,b = k$ (along the extension of $a\,b$); now, $b\,c + a\,b = a\,c = k + \sqrt{k^2 + 1}$; hence, $T_e = (a\,c)\,T$.

For solution of the Grashof formula, eq. (7), make $O\,a$ = unity, $O_1\,a = \tfrac{3}{4}$, and $O_2\,a = \tfrac{5}{4}$; lay off $O\,b = k$, hence $a\,b = \sqrt{k^2 + 1}$, and $a\,b_2 = \tfrac{5}{4}\sqrt{k^2 + 1}$. Now, since $O_1\,b_1 = \tfrac{3}{4}\,k$, if $b_2\,c$ be laid off equal to $O_1\,b_1$;

$$a\,c = b_2\,c + a\,b_2 = \tfrac{3}{4}\,k + \tfrac{5}{4}\sqrt{k^2 + 1}\,; \text{ hence } T_e = (a\,c)\,T.$$

Example 1. An engine is 16″ × 24″ (piston 16 inches in diameter and stroke of 24 inches), steam pressure = 100 pounds per sq. inch. The centre of the crank pin overhangs the centre of the main journal by 15″ (measured parallel to axis of the shaft). Assume that the pressure on the crank pin may be equal to 100 pounds unbalanced pressure per sq. inch of the piston when the connecting rod is perpendicular to the crank radius. Allowing 8000 pounds as the maximum fibre stress in the shaft; compute its diameter.

Area of piston = 200 sq. inches; radius of crank (arm of maximum twisting moment) = 12″; arm of bending moment = 15″.

$\therefore$ $T = 200 \times 100 \times 12 = 240{,}000$ inch pounds. Also, $k = 15 \div 12 = 1.25$.

$$T_e = \frac{f\,I_p}{r} = \frac{8000 \times \pi\,d^3}{16} = \left[1.25 + \sqrt{(1.25)^2 + 1}\right]240{,}000 =$$

$$\left[1.25 + 1.60\right]240{,}000 = 684{,}000 \text{ inch pounds.}$$

$$\therefore\ d^3 = \frac{684{,}000 \times 16}{8000\,\pi} = 435. \quad \therefore\ d = 7.58''.$$

17. **Combined Torsion and Compression.**—

Propeller shafts of steamers and vertical shafts carrying considable weight are subjected to combined twist and thrust. The span, or distance between bearings, is frequently so small that the shaft may be considered as subjected to simple compression, so far as the action of the thrust is concerned.

The intensity of this compressive stress is

$$f = \frac{4\,W}{\pi\,d^2},$$

in which $W =$ the thrust, and $d =$ the diameter of the (solid circular) shaft.

If $T =$ the twisting moment on the shaft, $r = \frac{1}{2}\,d =$ the radius of the shaft, $I_p =$ the polar moment of inertia $= 2\,I$ (the rectangular moment of inertia), and $f_s =$ the intensity of shearing stress due to T, then

$$T = \frac{f_s\,I_p}{r} = \frac{4 f_s\,I}{d} \quad \therefore\ f_s = \frac{T d}{4\,I} = \frac{16\,T}{\pi\,d^3},$$

for solid circular shafts.

The resultant maximum stress is that due to the combined actions of a normal stress (compression) and a tangential stress (shear), as in the case of combined bending and torsion (art. 16); hence, equation (2) of the preceding article may be used to find the equivalent, intensity of stress, or

$$f_n = \tfrac{1}{2} f + \tfrac{1}{2}\sqrt{f^2 + 4 f_s^2}$$

$$\therefore\ f_n = \frac{2\,W}{\pi\,d^2} + \frac{1}{2}\sqrt{\frac{16\,W^2}{\pi^2\,d^4} + \frac{4\,(16)^2\,T^2}{\pi^2\,d^6}} =$$

$$\frac{2\,W}{\pi\,d^2} + \frac{2}{\pi\,d^2}\sqrt{W^2 + \frac{64\,T^2}{d^2}} =$$

$$\frac{2}{\pi\,d^2}\left[\,W + \sqrt{W^2 + \frac{64\,T^2}{d^2}}\,\right] \qquad (1)$$

If the value of f_n is assigned, and d is to be found for given values of W and T, it is possible to solve the transformed (cubic) equation ; but it is much more convenient to assume a trial value of d (somewhat larger than that required for the twisting moment alone) and then to check for f_n.

If the span, between bearings, is so great that the shaft must be considered as a column liable to buckle, the value of the mean intensity of compressive stress, (p'), may be found by eq. (7) of art. 14.

Since p' is the mean intensity of compressive stress in the long column which corresponds to a maximum intensity of stress f, a short compression member of the same cross-section, would be capable of standing a load W_1 greater than W in the ratio of f to p' ; or $W_1 = \frac{f}{p'} W$. This value of W_1 is to be substituted for W in eq. (1) of this article, for shafts of long span.

III.

SPRINGS.

18. **Distinguishing Characteristic of Springs.**—Springs are characterized by a considerable distortion under a moderate load. Every machine member is, in a sense, a spring, for no material is absolutely rigid and the application of a load always produces stress and accompanying strain. By proper selection and distribution of material it is possible to control (within wide limits) the degree of distortion under a given load.

An absolutely rigid material would be practically unfit for the construction of any member subject to other than a perfectly quiescent load; for (as shown in art. 8) the stress due to a suddenly applied load would be infinite if the corresponding distortion of the member were zero.

While it is usually desirable to restrict the distortions of most machine parts to very small magnitudes, there are many cases in which considerable distortion under moderate load is desirable or essential. To meet this last requirement the member is often given some one of the forms commonly called springs.

19. **The Principal Applications of Springs.**—Springs are in common use:

I. For weighing forces; as in spring balances, dynamometers, etc.

II. For controlling the motions of members of a mechanism which would otherwise be incompletely constrained; for example, in maintaining contact between a cam and its follower. This constitutes what Reuleaux has called "force closure".

III. For absorbing energy due to the sudden application of a force (shock); as in the springs of railway cars, etc.

IV. As a means of storing energy, or as a secondary source of energy; as in clocks, etc.

An important class of mechanisms in which springs are used to weigh forces is a common type of governor for regulating the speed of engines or other motors. In those governors which use springs to oppose the centrifugal, or other inertia actions, the springs automatically weigh forces which are functions of speed, or of change of speed. The links, or other connections, which move relative to the shaft with any variation of the above forces, correspond to the indicating mechanism of ordinary weighing devices.

The first of the above mentioned applications—the weighing of forces—is usually the most exacting as to the relation between the load and the distortion of the spring throughout the range of action. In the second and third classes of application, it is frequently only required that the maximum load and distortion shall lie within certain limits, which often need not be very precisely defined. The use of springs for storing energy (as the term spring is ordinarily understood) is almost wholly confined to light mechanisms or pieces of apparatus requiring but little power to operate them.

20. **Materials of Springs.**—Springs are usually of metal; although other solid substances, as wood, are sometimes used. A high grade of steel, designated as spring steel, is the most common material for heavy springs, but brass (or some other alloy) is often used for lighter ones.

A confined quantity of air, or other compressible fluid, is used in many important applications to perform the office of a spring. The air-chamber of a pump with its inclosed air is a familiar example of what may be called a fluid spring used to reduce shock ("water hammer"). The characteristic distortion of the solid springs is a change in *form* rather than of volume; while the fluid springs are characterized by a change of *volume* with incidental change of form

Soft rubber cushions, or buffers, are not infrequently employed as springs, and these are in some respects intermediate in their action to the two classes mentioned above. It is usually not necessary, in these simple buffers, or cushions, to secure a very

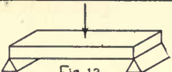


Fig. 13.

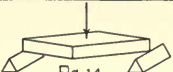


Fig. 14.

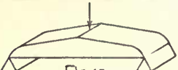


Fig. 15.

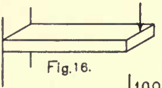


Fig. 16.

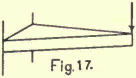


Fig. 17.

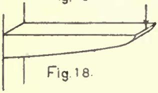
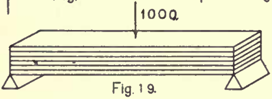


Fig. 18.



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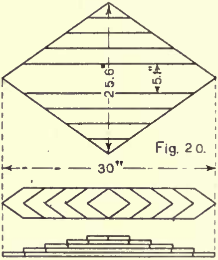


Fig. 20.

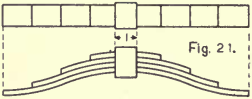


Fig. 21.

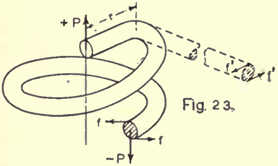


Fig. 23.

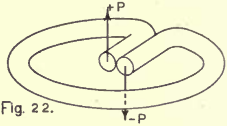


Fig. 22.

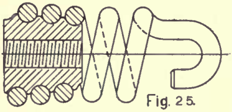


Fig. 25.

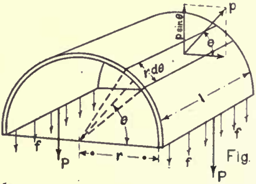


Fig. 26.

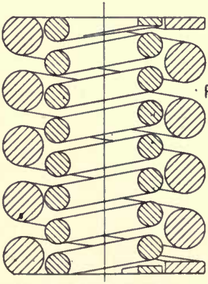


Fig. 24.

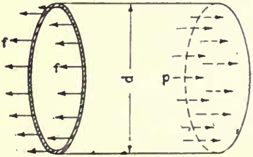


Fig. 27.

exact relation between the loads and the distortions under such loads. The discussion of the confined gases (fluid springs) is not within the scope of the present work; hence the following treatment will be limited to solid springs.

21. Forms of Solid Springs —Springs may be subjected to actions which extend, shorten, twist, or bend them, producing stresses in the material, the character of which depend upon both the form of the spring and upon the manner of applying the load.

I. *Flat Springs* are essentially beams, either cantilevers, or with more than one support. These springs are subjected to flexure when the load is applied, and the resultant stresses are tension in certain portions of the material, and compression in others, with a transverse shear, as in all beams; the shear may usually be neglected in computations. The ordinary beam formulas for strength and rigidity may be applied to flat springs, with constants appropriate to the particular material and form of beam used.

Flat springs may be simple prismatic strips, of uniform cross-section, (Figs. 13 or 16); although it is preferable that the form of such springs approximate those of the "uniform strength" beams (Figs. 14 or 15; 17 or 18).

It is often desirable or practically necessary to build up these springs of several layers, leaves, or plates, producing a laminated spring. It will appear from the discussion of these laminated springs that they may be properly treated as a modification of one form of "uniform strength" beam. The neutral surface of the beam used as a spring may be initially curved, either to clear other bodies, or to give the spring an advantageous form when it is under normal load. See Fig. 21.

Two or more springs may be compounded, as in the "elliptical" springs or in the platform springs frequently used under carriages. In such cases, each spring may be computed separately, and the total deflection is the sum of the deflections of the separate springs of the set.

II. *Helical, or Coil Springs* are most commonly used to resist actions which extend, shorten, or twist the spring relatively to

its longitudinal axis. These are sometimes improperly called spiral springs.

The stress in the wire (or rod) of which a helical spring is made is somewhat complex, consisting of torsion combined with tension or compression, or both. In a "pull spring", one which is extended longitudinally under the load, the predominating stress (with ordinary proportions) is a torsion, and there is a secondary tensile stress in the wire. In a "push spring", one which is shortened by the load, the predominating stress is torsion, with a secondary compressive stress. When the helical spring is subjected to an action which twists the spring (as a whole) the principal stress in the wire is that due to flexure (tension and compression in opposite fibres) and the secondary stress is torsion.

Helical springs are sometimes arranged in "nests", springs of smaller diameter being placed within those of larger diameter. In these cases, the different springs of a set are computed separately. This last arrangement is common practice in car trucks.

III. *Spiral Springs*, properly so called are those of the form of the familiar clock spring. These are best adapted for a twist relative to the axis of the spiral, and are usually employed when a very large angle of torsion between the two connections is necessary. In this form of spring, the stress in the material is that due to flexure; or tensile and compressive stress on opposite sides of the neutral axis.

IV. *Helico-Spiral Springs*. The form of spring represented by the common upholstery spring may be looked upon as a spiral spring which has been elongated, and given a permanent set, in the direction of its axis; or it may be considered as a modified helical spring in which the radii of the successive coils are not equal. It is thus intermediate between the two preceding classes. This last form is not usual in machine construction; though it has the advantage over the common helical spring of considerable lateral resistance, and it may be employed to advantage where it is difficult or undesirable to otherwise constrain the spring against buckling. This spring is only used as a push

spring, to resist a compressive action. The springs used on the ordinary disc valves of pumps are often of this form, as they will close up flat between the valve and guard. Car springs are sometimes made of a flat strip or ribbon of steel wound in this general form, with the edges of the strip parallel to the axis of the spring.

V. Occasionally straight rods, usually of circular or rectangular cross-sections, are employed to resist torsion relative to their longitudinal axis. These are comparatively stiff springs, and the stress is, of course, torsional. Every line of shafting is necessarily a spring, in this sense.

The following summary gives the ordinary forms of solid springs; the kinds of loading to which they are subjected; and the predominating stresses resulting from the different loads.

General Summary of Springs.

Form of Spring.	Load Action.	Predominating Stress.
Flat Spring.	Flexure or Bending.	Tension and Compression.
Helical Spring.	Extension, Pull.	Torsion (plus).
" "	Compression, Push.	Torsion (minus).
" "	Torsion, Twist.	Tension and Compression.
Spiral "	Torsion, Twist.	Tension and Compression.

22. Computations of Simple Flat Springs.—The following notation will be used in treating of flat springs with rectangular cross-sections:

P = load applied to the spring.

L = free length of the spring.

f = intensity of stress in outer fibres.

I = moment of inertia of most strained section.

h = dimension of this section in plane of flexure.

b = dimension of this section normal to plane of flexure.

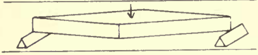
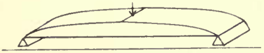
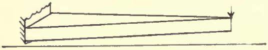
E = modulus of elasticity of material.

δ = deflection of the spring.

The six forms of rectangular section beams, shown by Figs. 13 to 18, are the most important of those used as simple flat springs.

These will be designated Type I, II, etc., as in the following table, which gives the constants to be substituted in the general formulas for computations relating to each type.

TABLE.

	Type.	Coefficients.				
		A	β	B	K	C
I		$\frac{2}{3}$	$\frac{1}{48}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{3}{2}$
II		$\frac{2}{3}$	$\frac{1}{32}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{3}{2}$
III		$\frac{2}{3}$	$\frac{1}{24}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{3}{2}$
IV		$\frac{1}{6}$	$\frac{1}{3}$	4	$\frac{2}{3}$	6
V		$\frac{1}{6}$	$\frac{1}{2}$	6	1	6
VI		$\frac{1}{6}$	$\frac{2}{3}$	8	$\frac{4}{3}$	6

The theory of strength against flexure gives: For rectangular section beams supported at the ends and loaded at the middle, (Types I, II, III).

$$\frac{1}{4} PL = \frac{1}{6} f b h^2 \therefore PL = \frac{2}{3} f b h^2 \qquad (1)$$

For rectangular section cantilevers, with load at free end,

$$PL = \frac{1}{6} f b h^2 \qquad (2)$$

Or the general formula for the strength of rectangular section beams may be written

$$PL = A f b h^2 \qquad (3)$$

In which the coefficient A has the values given in the Table.

The theory of elasticity of beams gives

$$\delta = \beta \frac{PL^3}{EI}, \tag{4}$$

or for rectangular cross sections

$$\delta = B \frac{PL^3}{Ebh^3} \tag{5}$$

In which β and B are as given in the Table, for the types under consideration.

The last equation (5) may be used for all computations as to rigidity of flat springs (beams), provided the elastic limit is not exceeded. The only constant for the *material* which enters this expression is the modulus of elasticity (E); this is simply the *ratio* of stress to strain which holds up to, but not beyond, the elastic limit; hence any computation made by this formula should be checked for safety. Equation (3) may be used for this purpose. To illustrate, assume that a rectangular section prismatic spring (Type I) has a length between supports of $L = 30''$; the load at the middle is $P = 1000$ lbs.; the deflection under this load is to be $\delta = 1.5$ inches; and the spring is made of a single strip of steel $\frac{3}{8}$ inch thick (h). Required the breadth (b) of the spring, assuming the modulus of elasticity, $E = 30{,}000{,}000$.

From eq. (5):—

$$b = B \frac{PL^3}{E\delta h^3} = \frac{1}{4} \times \frac{1{,}000 \times 27{,}000 \times 512}{30{,}000{,}000 \times 1.5 \times 27} = 2.84 + \text{ inches.}$$

This gives the width of spring for the required relation of the deflection to load; that is, it gives a spring of the required stiffness, provided the stress produced does not exceed the elastic limit. It is necessary to check the spring as found above, for if the elastic stress is passed, the spring not only takes a permanent set, but the required ratio of the load to the deflection will not be secured. On the other hand, it is often important for economy of material to use as light a spring as is consistent with safety; or in other words, it is important not to have too low a working stress under the maximum load.

From eq. (3):—

$$f = \frac{PL}{Abh^2} = \frac{3 \times 1000 \times 30 \times 64}{2 \times 2.84 \times 9} = 112{,}500 \text{ lbs. per sq. inch.}$$

This stress is beyond the elastic limit of any ordinary grade of steel, hence it is probable that some different form of spring should be used. A change could be assumed, as in the thickness of the plate, and new computations made with the new data. A thinner plate would reduce the stress, but it would demand a wider spring for the required stiffness. A more general method will now be given, by which it is possible to determine the proper spring for given requirements without the necessity of successive trial computations.

From eq. (3) :—

$$b h^2 = \frac{PL}{Af} \therefore b h^3 = \frac{PLh}{Af} \tag{6}$$

From eq. (5) :—

$$b h^3 = \frac{BPL^3}{E\delta} \tag{7}$$

From eqs. (6) and (7) :—

$$\frac{PLh}{Af} = \frac{BPL^3}{E\delta};$$

$$\therefore h = AB\frac{fL^2}{E\delta} = K\frac{fL^2}{E\delta} \tag{8}$$

From eq. (3) :—

$$b = \frac{1}{A}\frac{PL}{fh^2} = C\frac{PL}{fh^2} \tag{9}$$

The two equations (8) and (9) are in convenient form for designing a flat spring when the span (L), deflection (δ), load (P), and the material are given. Example : The span of a rectangular section prismatic flat spring (Type I) is 30 inches ; and a load of 1000 lbs. applied at the middle is to cause a deflection of 1.5 inches.

If the modulus of elasticity be 30,000,000 and the safe maximum working stress be taken at 50,000 lbs. per sq. in.,* required the dimensions of the cross section, h and b.

*If the spring is provided with stops to prevent deflection beyond a certain amount, the stress due to such deflection may be nearly equal to the elastic limit of the material. A very small factor of safety is all that is necessary.

From eq. (8) :—

$$h = K\frac{fL^2}{E\delta} = \frac{1}{6}\times\frac{50,000\times 900}{30,000,000\times 1.5} = \frac{1}{6}\text{ inch.}$$

Taking $h = \frac{5}{32}$ inch, to use a regular size of stock, f will be somewhat less than 50,000, or

$$f : 50,000 :: \tfrac{5}{32} : \tfrac{1}{6}\ ;\ \therefore f = 47,000.$$

From eq. (9) :—

$$b = C\frac{PL}{fh^2} = \frac{3}{2}\times\frac{1000\times 30\times \overset{1024}{\cancel{256}}}{47,000\times 25} = \overset{39.9}{\cancel{9.97}}\text{ inches.}$$

If this width is inadmissable, a laminated or plate spring may be used. See next article.

It will be noted that equation (8) does not directly involve either the load P or the breadth of spring b. It is evident that if a beam (flat spring) of given span (L), and thickness (h), is caused to deflect a given amount (δ), the outer fibres will undergo a definite strain which is not dependent upon the width of the beam (b), nor upon the force required to produce this change in relative positions of the molecules. As the unit strain multiplied by the modulus of elasticity equals the unit stress, it follows that this stress may be computed from L, h, and δ (which determine the strain), in connection with E. If the breadth of the beam (b) is increased, the force (P) required to produce the given deflection (δ) will be proportionately increased, but the intensity of stress is not affected by these changes alone.

This same conclusion may be reached from the following relation,* in which $\rho =$ the radius of curvature due to load.

$$\rho = \frac{EI}{\underset{M}{\cancel{PL}}} = EI \div \frac{fI}{\frac{1}{2}h} = \frac{Eh}{2f} \tag{10}$$

$$\therefore f = \frac{Eh}{2\rho} \tag{11}$$

It appears from eq. (11) that the stress is simply proportional to the thickness (h) and the radius of curvature (ρ), for any given

*See Church's Mechanics, page 261, and Unwin, page 51.

value of E. The span L, and the deflection δ, determine ρ, so that eq. (10) or (11) may take the place of eq. (8). Equations (10) and (11) are important in connection with the design of laminated springs.

23. **Laminated, or Plate, Springs** —It was shown in the preceding article that the maximum thickness of a simple flat spring is fixed when the span, deflection, and modulus of elasticity are known, and the intensity of working stress has been assigned. [See eq. (8).] With the value of the thickness (h) thus limited it will frequently happen that a simple spring will require excessive breadth (b) to sustain the given load, and it is often necessary to use a spring built up of several plates or leaves.

Example: $P = 1{,}000$ lbs; $L = 30''$; $f = 60{,}000$ lbs. per sq. in.; $\delta = 2''$, and $E = 30{,}000{,}000$. A simple prismatic spring, rectangular section, with load at the middle of the span (Type I), to meet the above requirements would have:

$$h = K\frac{fL^2}{E\delta} = \frac{1}{6} \times \frac{60{,}000 \times 900}{30{,}000{,}000 \times 2} = .15 \text{ inch.}$$

$$b = C\frac{PL}{fh^2} = \frac{3}{2} \times \frac{1000 \times 30}{60{,}000 \times .0225} = 33\tfrac{1}{3} \text{ inches.}$$

This spring, consisting of a plate .15 inch thick and $33\frac{1}{3}$ inches wide, with a span of 30 inches, is evidently an impracticable one for any ordinary case. Suppose this plate be split into six strips of equal width, each $33.3 \div 6 = 5.5''$ wide, and that these strips are piled upon each other as in Fig. 19; then, except for friction between the various strips, the spring would be exactly equivaleut, as to stiffness and intensity of stress, to the simple spring computed above. While the form of laminated spring which has just been developed might answer in some cases, another form, based upon the "uniform strength" beam (Type II), is much better for the ordinary conditions. It may be developed as follows, taking the same data as the preceding example except that the spring is to be of Type II (Table, page 44).

In the simple spring, Type II,

$$h = K\frac{fL^2}{E\delta} = \frac{1}{4} \times \frac{60{,}000 \times 900}{30{,}000{,}000 \times 2} = .225 \text{ inches.}$$

$$b = C\frac{PL}{fh^2} = \frac{3}{2} \times \frac{1{,}000 \times 30}{60{,}000 \times .0506} = 14.8 \text{ inches.}$$

A laminated spring for the case under consideration may be derived from this simple spring by imagining the lozenge shaped plate to be cut into strips which are piled one upon another as indicated in Fig. 20. The thickness .225 inches does not correspond to a regular commercial size of stock, however, and it will usually be better to modify the spring to permit using standard stock. If a thickness of $\frac{1}{4}''$ be assumed for the leaves, or plates, the stress, as found from eq. (8) of the preceding article becomes:

$$f = \frac{hE\delta}{KL^2} = \frac{4 \times .25 \times 30{,}000{,}000 \times 2}{900} = 66{,}700.$$

If this stress is considered too great, we might use steel $\frac{3}{16}''$ thick, when $f = \frac{4 \times 3 \times 30{,}000{,}000 \times 2}{16 \times 900} = 50{,}000.$

With $h = \frac{3}{16}''$, and $f = 50{,}000$,

$$b = C\frac{PL}{fh^2} = \frac{3}{2} \times \frac{1{,}000 \times 30 \times 256}{50{,}000 \times 9} = 25.6''.$$

If this spring, 30″ span, $\frac{3}{16}''$ thick, and 25.6″ wide at the middle, be replaced by 5 equivalent strips, each $25.6 \div 5 = 5.11''$ wide (nearly $5\frac{1}{8}''$), see Fig. 20, a laminated spring of good form and practicable dimensions will result. In cases where the maximum allowable width of spring is fixed, a larger number of plates may be necessary. Thus, in the preceding problem, if the spring width must be kept within $4\frac{1}{2}''$, it is necessary to use 6 plates, each $25.6 \div 6 = 4.27''$ wide. In actual springs, the usual construction is that shown by Fig. 21, in which the several plates have the ends cut square across instead of terminating in triangles. These springs approximate uniform strength beams, and

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may be computed by equations (8) and (9) of art. 22, remembering that b is the breadth of the equivalent simple spring. Or, if n is the number of plates and b_1 the breadth of each plate in the laminated spring, $nb_1 = b$.

The last of these formulas, eq. (9), is not strictly applicable when the ends of the plates are cut square across; but it may generally be used with sufficient accuracy, provided the successive plates are regularly shortened by uniform amounts. It is quite common practice to have two or more of the plates extend the full length of the spring. This construction makes the spring a combination of the triangular and prismatic types; (Type II and Type I, or Type V and Type IV, depending upon whether the spring is supported at the ends, or is a cantilever). Mr. G. R. Henderson in discussing the cantilever form, (Trans. A. S. M. E. vol. XVI) says:—"For a spring with all the plates full length we would have

$$\delta = \frac{4\,PL^3}{Enb_1h^3}$$

so for one-fourth of the leaves full length, the deflection would be decreased approximately one-fourth of the difference between

$$\frac{6\,PL^3}{Enb_1h^3} \text{ and } \frac{4\,PL^3}{Enbh^3} \text{ or } \frac{5.5\,PL^3}{Enbh^3}\,."$$

By similar reasoning, for a spring loaded at the middle and supported at the ends, with one-fourth the plates extending the whole length of the spring,

$$\delta = \frac{11}{32}\,\frac{PL^3}{Enb_1h^3}\,.$$

This may be otherwise stated as follows:

When the number of full length leaves is one-fourth the total number of leaves in the spring, use $\frac{11}{12}B$ instead of B and $\frac{11}{12}K$ instead of K in equations (5) and (8) of the preceding article; the value of K being that given for the triangular forms, Type II or Type V, as the case may be.

The spring shown in Fig. 21 is initially curved (when free), which is common practice. The best results are obtained by having the plates straight when the spring is under its normal full load (if this is practicable) because the sliding of the plates upon each other, with the vibrations, is then reduced to a minimum.

The several plates of a laminated spring are usually secured by a band shrunk around them at the middle of the span. This band stiffens the spring at the middle, and ½ the length of the band (½ l, Fig. 21) may be deducted from the full span to give the effective span to be used as L in the above formulas. It is not uncommon to make the longest plate thicker than the others, if but one plate is given the full length of the spring. This cannot be looked upon as desirable practice, however, as all of the plates are subjected to the same change in radius of curvature; hence the thicker plate is subjected to the greater stress. See eq. (11), art. 22.

The following formulas (derived from the preceding) may be used in computing flat springs; but it must be remembered that there is always liability of considerable variation in the modulus of elasticity, hence such computations can only be expected to give approximations to the deflections which will be observed by tests of actual springs. These computations will be sufficiently exact for many purposes; but when it is important to accurately determine the scale of the spring (ratio of deflection to load), actual tests must be made. In using these formulas the following rules should be observed.

I. When the several plates are secured by a band shrunk, or forced, over them, one-half the length of the band is to be subtracted from the length of the spring to get the effective length of the spring.

II. When the plates have different thicknesses, the stress should be computed from the plate having the maximum thickness.

III. If more than one plate has the full length of the spring, an appropriate modification of the values of the coefficients B and

and K should be made. Thus, when one-fourth of the total number of plates are full length, $\frac{11}{12} B$ and $\frac{11}{12} K$ should be used instead of B and K (Type II or V) in equations I, II, III, and IV.

EQUATIONS.

$$\delta = B \frac{P L^3}{E n b_1 h^3} \tag{I}$$

$$P = \frac{E n b_1 h^3 \delta}{B L^3} \tag{II}$$

$$h = A B \frac{f L^2}{E \delta} = K \frac{f L^2}{E \delta} \tag{III}$$

$$f = \frac{E \delta h}{K L^2} \tag{IV}$$

$$f = \frac{E h}{2 \rho} \tag{V}$$

$$P = \frac{A f n b_1 h^2}{L} \tag{VI}$$

$$f = \frac{C P L}{n b_1 h^2} \tag{VII}$$

$$b = \frac{P L}{A n f h^2} = \frac{C P L}{n f h^2} \tag{VIII}$$

Experience shows that thin plates have a higher elastic limit than thick plates of similar grade of material. In the practice of a prominent eastern railway company, the values allowed for the maximum intensity of stress in flat steel springs are, for :

Plates	$\frac{1}{4}$	inch	thick,	$f = 90{,}000$	lbs. sq. in.
"	$\frac{5}{16}$	"	"	$f = 84{,}000$	" "
"	$\frac{3}{8}$	"	"	$f = 80{,}000$	" "
"	$\frac{7}{16}$	"	"	$f = 77{,}000$	" "
"	$\frac{1}{2}$	"	"	$f = 75{,}000$	" "

The above values are satisfied by the equation $f = 60{,}000 + \frac{7{,}500}{h}$, in which h is the thickness of plate.

These values are for the greatest stress to which the material can be subjected, as when the spring is deflected down against the stops.

The modulus of elasticity, E, may vary considerably; but its value may be assumed at about 30,000,000 in the absence of more definite data.

In designing a new spring, the value of h is to be found from equation (III); then b is found by equation (VIII). The other formulas are useful in checking springs already constructed for deflection due to a given load, or the reverse; for safety, etc.

24. **Helical Springs.**—[Unwin, § 34a, pages 72, 73.] If a rod or wire be wound into a flat ring with the ends bent in to the centre, Fig. 22, and two equal and opposite forces, $+P$ and $-P$, be applied to these ends (perpendicular to the plane of the ring) as indicated, the rod will be subjected to torsion.

If a longer rod be wound into a helix, with the two ends turned in radially to the axis, the typical helical spring is produced. If two equal and opposite forces, $+P$ and $-P$, act on these ends, along the axis of the helix, they induce a similar stress (torsion) in the rod, but as the coils do not lie in planes perpendicular to the line of the forces, there is a component of direct stress along the rod. This direct stress increases as the pitch of the coils increases relative to their diameter; but with ordinary proportions of springs, the torsion alone need be considered, when the external forces lie along the axis of the helix.

The following notation will be used in treating of helical springs of circular wire, subjected to an axial load:

$P =$ the force acting along the axis.
$r =$ the radius of the coils, to centre of wire.
$d =$ the diameter of wire.
$f =$ the maximum intensity of stress in wire (torsion).
$I_p =$ the polar moment of inertia of wire.
$G =$ the transverse modulus of elasticity.
$\delta =$ the "deflection" (elongation or shortening) of spring.
$n =$ the number of coils in the spring.
$L =$ the length of wire in the helix $= 2\pi r n$ (approximately).

Suppose a helical spring under an axial load to be cut across the wire at any section, and the portion on one side of this section to be considered as a free body, Fig. 23. Neglecting the direct stress, equilibrium demands that the moment of the external force (Pr) shall equal the stress couple, or moment of resistance, $\left(\frac{\pi}{16} f d^3 \text{ for circular section }\right)$.

If this free portion of the helix is straightened out, as indicated by the broken lines in Fig. 23, till its direction is perpendicular to the radial end, it will appear that the moment Pr still equals the moment of resistance, $\frac{\pi}{16} f d^3$. Since the stress and strain are the same in this helix and the straight rod, it appears that the energy expended against the resilience is the same in both cases (the length of wire affected remaining constant). Or, as the force (P) and the arm (r) are the same in both conditions, the distances through which this force acts to produce a given torsional stress (f) are equal. If a straight rod of length L is subjected to a torsion moment Pr, the angle of twist being α (in π measure),

$$Pr = \frac{\alpha I_p G}{L}$$

[See Church's Mechanics, page 236.]

The energy expended on the rod is the mean force applied multiplied by the distance through which this force acts If the load is gradually applied, this energy is $\frac{1}{2} Pr\alpha$. In the case of the corresponding helical spring, the mean force $(\frac{1}{2} P)$ acts through a distance equal to the "deflection" of the spring (δ), or the energy expended is $\frac{1}{2} P\delta$. As pointed out above, the energy expended in the two cases is the same, or

$$\tfrac{1}{2} Pr\alpha = \tfrac{1}{2} P\delta \quad \therefore \quad \alpha = \frac{\delta}{r}$$

$$\therefore \quad Pr = \frac{\alpha I_p G}{L} = \frac{\delta}{r} \times \frac{\pi d^4}{32} \times \frac{G}{2\pi r n} = \frac{\delta d^4 G}{64 r^2 n}$$

$$\therefore \quad P = \frac{\delta d^4 G}{64 r^3 n} \qquad (1)$$

Equation (1) may be used for finding the load corresponding to an assigned deflection in a given spring. The equation can be put in the following form for finding the deflection due to a given load:

$$\delta = \frac{64\,P r^3 n}{d^4\,G} \tag{2}$$

Or the equation may be employed for designing a spring in which the load and deflection are given, by assuming any two of the three quantities, r, d and n. The most convenient form for this latter purpose is usually,

$$n = \frac{\delta\, d^4\, G}{64\, P r^3} \tag{3}$$

These equations for *rigidity* only hold good within the elastic limit of the material, as G is simply a *ratio* between stress and strain within this limit. It therefore becomes necessary to check any of the above indicated computations for strength, and it will often be found, after thus checking, that the stress is either too high for safety, or too low for economy.

The formula for the *strength* of a solid circular-section rod under torsion is

$$\left.\begin{aligned} P r &= \frac{\pi}{16} f d^3 \quad \therefore \quad P = \frac{\pi f d^3}{16\, r}\,; \\ r &= \frac{\pi f d^3}{16\, P}\,; \qquad f = \frac{16\, P r}{\pi d^3} \end{aligned}\right\} \tag{4}$$

It is to be remembered that as equation (4) is for safe strength, the load (P) should be the maximum load to which the spring can be subjected; but equation (3) may be used with any load and the corresponding deflection.

Example: The load on a helical spring is 1600 lbs., and the corresponding deflection is to be 4″. Transverse modulus of elasticity of material = 11,000,000, and the maximum intensity of safe torsional stress = 60,000 lbs. wire of circular section. To design the spring, assume $d = \frac{5}{8}''$, and $r = 1\frac{1}{2}''$; from eq. (3),

$$n = \frac{4 \times 625 \times 11{,}000{,}000 \times 8}{4096 \times 64 \times 1600 \times 27} = 19.4$$

Checking for the stress by the last equation in group (4),

$$f = \frac{16 \times 1600 \times 1.5 \times 512}{\pi \times 125} = 50{,}200 \text{ lbs.}$$

This stress is found to be safe, but is considerably below the limit assigned, and it may be desirable to work up to a somewhat higher stress. Another computation can be made (with a smaller d or larger r), and by a series of trials, the desired spring can be found. The following order of procedure avoids this element of uncertainty. The load being given, assume a diameter of wire and value of safe stress, then solve in eq. (4) for the radius of coil. Make this radius some convenient dimensions (not exceeding that computed if the assumed stress is considered the maximum safe value). Next substitute these values of d and r (with those given for P, δ and G) in eq. (3) to find the number of coils. Thus, with the data of the preceding example, assuming $d = \frac{5}{8}''$;

$$r = \frac{\pi}{16}\frac{f d^3}{P} = \frac{\pi \times 60{,}000 \times 125}{16 \times 1600 \times 512} = 1.79''$$

If the $\frac{5}{8}''$ rod is wound on an arbor $3''$ diameter, the radius to the centre of coils will be about $1.81''$; and the corresponding stress would be 60,500 lbs. per square inch. This is so slightly in excess of the assigned value that it may be permitted, especially as this value is a moderate one for spring steel. Substituting in eq. (3),

$$n = \frac{\delta d^4 G}{64 P r^3} = \frac{4 \times 11{,}000{,}000 \times 625}{64 \times 1600 \times 5.93 \times 4096} = 11.1.$$

It may be desirable to fix upon the radius of coil, rather than the diameter of wire, in the first computation, in designing a spring. From eq. (4):

$$d^3 = \frac{16 P r}{\pi f}, \quad \therefore \quad d = 1.72 \sqrt[3]{\frac{P r}{f}} \qquad (5)$$

In other cases, it may be desirable to assume the ratio of the radius of coil to the diameter of wire, then from eq. (4):

$$d^2 = \frac{16 P}{\pi f} \frac{r}{d}, \quad \therefore \quad d = 2.26 \sqrt{\frac{P}{f}\left(\frac{r}{d}\right)} \tag{6}$$

In either of the preceding conditions, use a regular size of wire.

In checking a given spring, it may be required to determine either the safe load, or the safe deflection. If the former is the case, eq. (4) may be used directly. If it is required to find the safe deflection, substitute the value of P from eq. (4) in eq (2) and the result is

$$\delta = \frac{12.57 \, n r^2 f}{G d} \tag{7}$$

The weight of a spring is a matter of some importance, as the material is expensive. The following discussion shows that the weight varies directly as the product of the load and the deflection, inversely as the square of the intensity of stress in the wire, and directly as the transverse modulus of elasticity. Hence for a given load and deflection, economy calls for a high working stress and a low modulus of elasticity. From eq. (4):

$P = \frac{\pi}{16} f \frac{d^3}{r}$; also for a member under torsion,

$f = \frac{d}{2} \times \frac{\alpha G}{L}$ [Church's Mechanics, p. 235].

$$\therefore \quad f = \frac{d}{2} \times \frac{\delta}{r} \times \frac{G}{2 \pi r n} = \frac{d \delta G}{4 \pi r^2 n}$$

$$\therefore \quad \delta = \frac{4 \pi r^2 n f}{d G} \tag{8}$$

$$\therefore \quad P\delta = \frac{\pi^2 d^2 r n f^2}{4 G} \tag{9}$$

But the volume of the spring is

$$v = \tfrac{1}{4} \pi d^2 L = \tfrac{1}{2} \pi^2 d^2 r n \tag{10}$$

$$\therefore \quad P\delta = \frac{f^2 v}{2G}, \quad \therefore \quad v = \frac{2G}{f^2} P\delta \tag{11}$$

The weight is directly proportional to the volume; hence, for given values of G and f, the weight varies simply as the product of the load and the deflection. All possible helical springs (of similar section of wire) have the same weight for a given load and deflection, if of the same material and worked to the same stress. It can be shown that a helical spring of square wire must have 50 per cent. greater volume than one of round wire, the stress and modulus of elasticity being the same in both. The round section is generally admitted to be best for helical springs under ordinary conditions.

A small wire of any given steel usually has a higher elastic limit than a larger one, while there is not a corresponding change in the modulus of elasticity with change in diameter. This suggests the use of as light a wire as is consistent with other requirements.

An extensive set of tests of springs, conducted by Mr. E. T. Adams, in the Sibley College Laboratories, indicate that the steel such as is used in governor springs may be subjected to stress varying from about 60,000 lbs. per square inch with $\frac{3}{4}''$ wire to 80,000 lbs. per square inch (or more) in wire $\frac{3}{8}''$ diameter. The following expression may be used to find the safe stress in such springs:

$$f = 40{,}000 + \frac{15{,}000}{d}. \tag{12}$$

Mr. J. W. Cloud presented a most valuable paper on Helical Springs before the Am. Society of Mechanical Engineers (Trans. Vol. V, page 173), in which he shows that for rods used in railway springs ($\frac{3}{4}''$ to $1\frac{5}{16}''$ diam.) the stress may be as high as 80,000 lbs. per square inch, and that the transverse modulus of elasticity is about 12,600,000.

Two or more helical springs are often used in a concentric nest (the smaller inside the larger); all being subjected to the same deflection. This is common practice in railway trucks, where the springs are under compression when loaded. If these springs have the same "free" heighth (when not loaded), and if they are

of equal heighth when closed down "solid," Mr. Cloud shows that the length of wire should be the same in each spring of the set for equal intensity of stress. The "solid" heighth of a spring is $H = d\,n$, and the length of wire is $L = 2\,\pi\,r\,n$; hence the numbers of coils of the separate springs of the set are inversely as the diameters of the wire and inversely as the radii of the coils; or the ratio of r to d is the same in each spring of the nest. This conclusion may be somewhat modified when it is remembered that the wire of smaller diameter may usually be subjected to somewhat higher working stress than the larger wire of the outer helices; and also that the wire of these compression springs is commonly flattened at the end to secure a better bearing against the seats. See Fig. 24.

Two common methods of attaching "pull" springs are shown in Fig. 25. One end of the spring shows a plug with a screw thread to fit the wire of the spring. This plug is usually tapered slightly, and the coils of the spring are somewhat enlarged by screwing it in. The other end of the spring shows the wire bent inward to a hook which lies along the axis of the helix. The former method is usually preferable for heavy springs.

SUMMARY OF HELICAL SPRING FORMULAS.

$$P\,r = \frac{\pi}{16} f\,d^3 \qquad \text{(I)} \qquad\qquad \delta = \frac{12.57\,n\,r^2 f}{G\,d} \qquad \text{(VII)}$$

$$r = \frac{\pi f\,d^3}{16\,P} \qquad \text{(II)} \qquad\qquad P = \frac{\delta\,d^4\,G}{64\,r^3\,n} \qquad \text{(VIII)}$$

$$d = 1.27\sqrt[3]{\frac{P\,r}{f}} \qquad \text{(III)} \qquad\qquad \delta = \frac{64\,P\,r^3\,n}{d^4\,G} \qquad \text{(IX)}$$

$$d = 2.26\sqrt{\frac{P}{f}\left(\frac{r}{d}\right)} \qquad \text{(IV)} \qquad\qquad n = \frac{64\,P\,r^3\,n}{d^4\,G} \qquad \text{(X)}$$

$$f = \frac{16}{\pi}\,\frac{P\,r}{d^3} \qquad \text{(V)} \qquad\qquad v = \frac{f^2}{2\,G}\,P\,\delta \qquad \text{(XI)}$$

$$P = \frac{\pi f\,d^3}{16\,r} \qquad \text{(VI)}$$

Formulas (I) to (VII), inclusive, relate to strength ; (VIII) to (X), inclusive, relate to rigidity, or elasticity.

In the absence of more exact information as to the properties of the material of which a steel helical spring is made, the following values may be taken :

$$G = 12{,}000{,}000,$$

$$f = 40{,}000 + \frac{15{,}000}{d}.$$

25. Spiral or Helical Springs in Torsion—The following formulas for either true spiral or helical springs subjected to torsion are derived from "The Constructor," by Professor Reuleaux.

$$\phi = \frac{PRL}{EI}\ ;\ f = \frac{WR}{Z},$$

In which

P = load applied to rotate axle,
R = lever arm of this load,
ϕ = angle through which axle turns,
L = length of effective coils,
E = modulus of elasticity (direct),
I = moment of inertia of the section.

IV

PIPES, TUBES AND FLUES.

26. **Pipes, Tubes and Flues,** of wrought iron, steel and cast iron have many applications in mechanical constructions, and the requirements are quite different in various services. However, there are certain standard forms well adapted to varied uses.

Wrought iron or steel *pipes*, such as are used for steam, water, gas, etc., are designated by the nominal *inside* diameters ; while *tubes*, such as are used in boilers, are rated by the *outside* diameters. The process of making ordinary pipes and tubes is to roll a long strip into a tube somewhat larger in diameter than the finished size ; then to draw this tube, while at a welding heat, through reducing dies. This welds the edges together and reduces the pipe (or tube) to the required size. Small size pipes (usually up to 1 inch) are "butt welded" ; larger sizes are "lap welded."

There are numerous processes for making seamless tubes. These tubes are largely used for bicycle frames, and to some extent for other service. They are much stronger than welded tubes, as the latter usually fail, if at all, by splitting along the seam. The welded tube is much cheaper, however, and is safely used for most services.

Boiler tubes are thinner than pipes of similar diameter, because the latter are given sufficient thickness to permit cutting threads on the ends Owing to the process of manufacture, the outside diameter varies but little from the standard size, while any variation in thickness of metal produces variation of inside diameter. A two inch tube will be very nearly 2″ outside diameter, while an ordinary 2″ pipe is about $2\frac{3}{8}''$ outside diameter and $2\frac{1}{16}''$ inside diameter. Pipes usually have an actual inside diameter rather greater than the nominal size. This variation exceeds one-

eighth of an inch in certain sizes; while in a standard 2½″ pipe (and some few of the large sizes) the actual inside diameter is slightly less than the nominal dimension. Beside the standard pipes, which have an ample factor of safety against bursting under ordinary pressures, there are thicker pipes known as "extra strong", and "double extra strong." These latter are suitable for high pressures, as in connection with hydraulic machinery, etc. The "extra strong" and "double extra strong" pipes are drawn by dies the same diameter as those used for the same nominal sizes of standard pipes; hence the inside diameters are considerably less, owing to the extra thickness of walls.

The following actual dimensions illustrate:

TWO INCH PIPE.

	Outside Diam.	Inside Diam.
Standard, - -	2.375″	2.067″
Extra Strong, - .	2.375	1.933
Double Extra Strong, -	2.375	1.491

Various books and catalogues contain tables of dimensions of pipes and tubes.

Wrought iron and steel pipes are usually joined together by screwing the threaded ends into couplings (sleeves), or into flanges. The former method is most common in sizes up to about 6″, while bolted flanges are commonly used with larger sizes.

The Briggs Standard Pipe Threads, almost universally used in this country, have for

⅛″ pipe, 27 threads per inch.
¼″ and ⅜″ pipe, 18 threads per inch.
½″ and ¾″ pipe, 14 threads per inch.
1″ to 2″ pipe, 11½ threads per inch.
2½″ and over, 8 threads per inch.

For form of threads and other details as to Briggs system, see Trans. A. S. M. E., Vol. VIII, page 29.

There is much variation in the practice of various makers of fittings in the dimensions of pipe flanges. A committee of the

American Society of Mechanical Engineers suggested a standard (Trans., Vol. XIV, page 49), after considering the various requirements and existing practice. This system is by no means generally adopted as yet, however.

Cast iron pipes are generally used for water works systems. These are sometimes joined by flanges cast with the sections of pipe, but more frequently the joint is of the "bell and spigot" form. The "bell" is a cup-shaped enlargement on one end of the section, into which the smaller end of the adjacent pipe is inserted. After placing the "spigot" in the "bell" the annular space is calked with "oakum" or other fibrous material; then melted lead is poured in, and afterwards calked, to retain the soft packing. The spigot end is usually provided with a "bead" to make it less liable to work out This form of joint permits a certain degree of flexibility, not secured with bolted flanges, which is desirable if the ground settles unequally under the pipe.

Cast iron pipe should be cast on end (with the axis vertical) as this reduces the danger of producing an unsymmetrical pipe through springing of the core, or accumulation of dirt on one side of the casting.

Tubes are used in boilers either for conveying the hot gases through the water, (fire tubes), or for conducting the water through the hot gases (water tubes). Large tubular passages for the hot gases are called flues. These latter very often have riveted seams instead of welded ones.

If a pipe is used for conveying water (or other fluid) under heavy pressure, the primary straining action is a bursting one, which produces a tensile stress in the material. There is also liability of considerable shock ("water hammer") in many cases. With gases or dry vapors there is less liability of shock due to sudden change in the velocity of the flow than with liquids; but faulty drainage with pipes containing steam (or other vapors which condense readily) may result in most violent shock to the line. In gas mains, exhaust steam pipes, etc., the bursting pressure is often small. These lines may be in greater danger from such

irregular actions as unequal settling of the ground (if buried), or of the supports if carried otherwise; from temperature changes, etc. Expansion and contraction frequently result in most dangerous straining actions, unless properly provided for. Boiler tubes and flues may be permanently injured, or so temporarily weakened as to cause initial rupture, by becoming over heated through low water. Some of the above actions, for example expansion and contraction, may often be foreseen and proper provision be made for them; while such straining actions as unequal settling of supports, over-heating, etc., are less determinate.

27. Resistance of Thin Cylinders to Internal Pressure. [Unwin, § 26a, page 47.] If a cylinder of circular section, subjected to an internal unit bursting pressure (p), has a thickness (t) which is small relative to its diameter (d), the intensity of stress along any longitudinal section is $f=\frac{p\,d}{2\,t}$, as given by Unwin. Fig. 26 shows one half of such a cylinder as a free body. The normal pressure on a longitudinal strip of length l and width $r\,d\,\theta$ is $p\,l\,r\,d\,\theta$. The total stress on the two free sections (each of area $= t\,l$) is

$$2\,P=2\,f\,t\,l=\int_0^\pi p\,l\,r\,d\theta\sin\theta=p\,l\,r\int_0^\pi\sin\theta\,d\theta=2\,p\,l\,r=p\,d\,l$$

$$* \quad \therefore f=\frac{p\,d}{2\,t} \tag{1}$$

$$p=\frac{2\,f\,t}{d} \tag{2}$$

$$t=\frac{p\,d}{2\,f} \tag{3}$$

If a transverse section of the cylinder be considered, (Fig. 27), it will be seen that the total pressure on the head, which tends

*The result would have been the same if the length of the shell had been treated as unity, because the total stress and the area over which this stress is distributed both vary directly as the length, or l cancels out in any case.

to cause rupture along a transverse section, is $\frac{\pi}{4} d^2 p$; and this is equal to the intensity of stress produced multiplied by the area of metal in such a section or

$$\frac{\pi}{4} d^2 p = \pi d t f$$

$$\therefore \qquad f = \frac{p d}{4 t} \qquad (4)$$

$$p = \frac{4 t f}{d} \qquad (5)$$

$$t = \frac{p d}{4 f} \qquad (6)$$

A comparison of (1) and (4) shows the stress in transverse sections to be only one-half that in longitudinal sections. For this reason it is very common practice to make circumferential seams of a boiler shell single riveted, when the longitudinal seams are double riveted.

A comparison of (2) and (5) shows that for a sphere (all of the sections of which correspond to transverse sections of a cylinder) the pressure is twice as great as in a cylinder of the same thickness and diameter for the same maximum stress.

In a cylinder (pipe or flue) which has a riveted seam, the computations should, of course, be made for a section passing through such seam. The ratio of the strength of the section through the seam to the strength of a parallel section through the solid plate is called the "efficiency of the joint" (ϵ). The method of computing the efficiency for any form of riveted joint will be given in the next chapter; but it is always less than unity and may, when known, be used as follows:

The stress in the solid plate equals the stress at the joint multiplied by the efficiency; then if f is the stress allowed at the weakened sections, ϵf is the stress in the solid plate or, for the longitudinal sections,

$$\therefore \qquad \epsilon f = \frac{p\,d}{2\,t}$$

$$f = \frac{p\,d}{2\,t\,\epsilon} \qquad (1a)$$

$$p = \frac{2\,\epsilon f\,t}{d} \qquad (2a)$$

$$t = \frac{p\,d}{2\,\epsilon f} \qquad (3a)$$

28. Resistance of Non-Circular thin Cylinder to Internal Pressure.—Suppose a cylinder to have a cross-section made up of circular arcs, as in Fig. 28. Take the upper half as a free body (section along the major axis). Let the resultants of the components of pressure which are normal to the plane of the section be P_1, P_2, and P_3 for the portions marked I, II, and III, respectively. Then these resultant forces, per unit of length of the cylinder, are as follows.

$$P_1 = p\,r \int_0^{\phi'} \sin\phi\,d\phi = p\,r\,(-\cos\phi' + \cos 0) = p\,m_1\,;$$

$$P_2 = p\,R \int_{\theta''}^{\theta'} \sin\theta\,d\theta = p\,R\,(-\cos\theta' + \cos\theta'') = p\,m_2\,;$$

$$P_3 = p\,r \int_{\phi''}^{\pi} \sin\phi\,d\phi = p\,r\,(-\cos\pi + \cos\phi'') = p\,m_3.$$

$$\therefore P_1 + P_2 + P_3 = p\,(m_1 + m_2 + m_3) = p\,A.$$

In a similar way, if the section is taken along the minor axis, the resultant force normal to this axis is found to be $p\,B$. In like manner the resultant force normal to any section is (per unit of length of cylinder) equal to the intensity of pressure multiplied by the axis of that section. As B is less than A, the resultant force $p\,B$ is less than $p\,A$; or the force tending to elongate the minor axis is greater than the force tending to elongate the major axis. If the tube were perfectly flexible, its form of cross-section would become, under pressure, one in which all axes are equal,

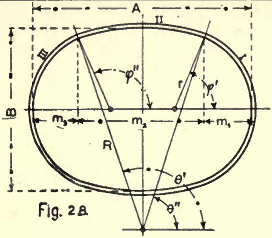

Fig. 28.

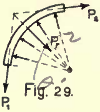

Fig. 29.

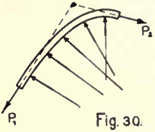

Fig. 30.

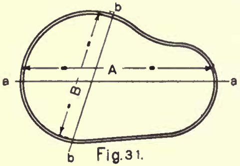

Fig. 31.

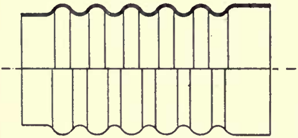
Fig. 33.

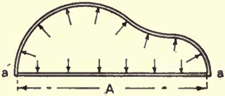

Fig. 32.

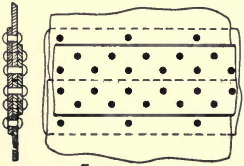
Fig. 34.

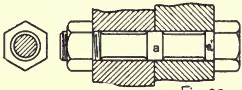

Fig. 35.


Fig. 36.

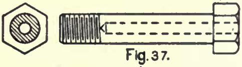
Fig. 37.

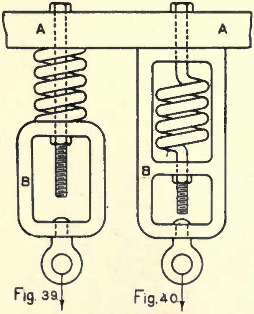

Fig. 39. Fig. 40.

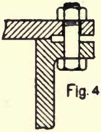
Fig. 41.

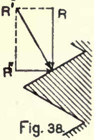

Fig. 38.

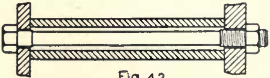
Fig. 42.

or circular. A rigid material offers resistance to such change of form ; and a flexural stress is produced in addition to the direct tension, but it approaches nearer to the circular form as the pressure increases. The existence of this flexure stress in a non-circular cylinder becomes apparent from a comparison of Figs. 29 and 30 In Fig. 29 (circular section) the lines of normal pressure all pass through a single point (the centre of the circle) ; the resultant (P_r) of the tensions (P_1 and P_2) also passes through this same point, hence, these forces form a concurrent system, and they are in equilibrium. In Fig. 30, however, the pressures do not in themselves form a concurrent, nor parallel, system of forces, hence, they cannot be balanced by a single force (as the resultant P_r), but there must be a moment, or moments, of stress for equilibrium. A similar course of reasoning could be applied to a cylinder of any non-circular cross section ; for such a section (Fig. 31) could be considered as made up of circular arcs, each of which could be treated (like the special case of Fig. 28) by integrating between proper limits. A direct inspection will also show that in any non-circular section cylinder, subjected to internal pressure, the pressure tends to reduce the cylinder to a circular cross-section. Suppose the original cylinder (Fig. 31) to be cut along the greatest axis of its cross-section, and that a flat bottom, coinciding with this section-plane be secured to it, as in Fig. 32. The total pressure on this bottom evidently balances the components of the pressure on the curved surface which lie normally to this flat bottom ; hence, the resultant of these normal components of pressure equals $p(a\,.\,.\,a) = pA$, per unit of length of cylinder. In a similar way, the resultant of components of pressure acting normally to any other section, (as $b\,.\,.\,b$, Fig. 31) equals $p(b\,.\,.\,b) = pB < pA$. This direct method might have been used in the preceding cases (Figs. 26 and 28) without recourse to the calculus.

It is apparent, then, that any cylinder under internal pressure tends to assume a circular cross-section A cylinder of nominal circular section, but departing from the true form to some extent, tends to correct this departure under internal pressure ; or if a

circular cylinder under internal pressure is deformed by any external force, it tends to resume its circular shape. Thus a circular cylinder under internal pressure is in "stable equilibrium." If the section is other than a true circle there is a flexure stress, as well as tension, when under pressure.

29. **Resistance of Thick Cylinders under Internal Pressure.**—[Unwin, § 26 a, page 48]. If the walls of the cylinder are quite thick relative to its diameter, the intensity of stress is not uniform across any section. The inner layers are strained most, because it is by yielding of the inner layers that stress is induced in the outer layers. The Grashof formulas. Unwin, page 48, equations (3) and (3a), may be used for thick cylinders.

Equation (3) is to be used in the general case. For convenience it is given here.

$$t = \frac{d}{2}\left\{-1 + \sqrt{\frac{3f + 2p}{3f - 4p}}\right\} \qquad (1)$$

Example :—The cast iron cylinder of a hydraulic press is 8″ diameter ; intensity of fluid pressure is to be 1000 lbs. per sq. inch ; safe tensile stress in material taken at 2,500 lbs. per sq. inch. Required the thickness of walls. By eq. (3),

$$t = \frac{8}{2}\left\{-1 + \sqrt{\frac{3 \times 2{,}500 + 2 \times 1{,}000}{3 \times 2{,}500 - 4 \times 1{,}000}}\right\} = 2.6 \text{ inches.}$$

The thin cylinder formula,—eq. (3), art. 27,—if applied to this problem, would give a thickness of only 1.6 inches.

An examination of the above formula shows that if $4p = 3f$ ($p = \frac{3}{4}f$) the thickness would be infinite. This does not mean that a pressure of 1875 pounds per square inch would necessarily burst the cylinder, in the preceding example, but it indicates that this pressure would produce a stress exceeding 2,500 lbs. maximum intensity, in a cylinder of 8 inches diameter, however thick it may be.

30. **Resistance of Thin Cylinders to External Pressure.**—[Unwin, § 40–41, page 82–85.] A similar analysis to that given for cylinders under internal pressure (art. 27,) could be applied to

this case. As the normal pressures are reversed in direction the stresses produced are of opposite sign to those of the former case; or the stress in the walls of the cylinder becomes compression under the application of external pressure.

If the non-circular cylinders of either Figs. 28 or 31 be considered as subjected to external pressure, the force tending to increase the major axis will be seen to be greater than those tending to increase any shorter axis; hence, the external pressure will cause collapse of the cylinder, unless the flexural rigidity of the material is sufficient to prevent this action. In a cylinder of nominal circular section, any departure from the ideal section would be increased by the external pressure: Or, if a cylinder of true circular section is deformed in any way while under external pressure, this pressure would tend to still further increase the deformation. In other words, the cylinder under external pressure is in "unstable equilibrium." As perfectly true circular sections and homogeneous materials are not attainable under the conditions of practice, the danger of collapse must often be taken into account in designing a pipe, tube or flue, to withstand external fluid pressure. The resistance of the heads or the flanges at the ends reinforces the shell against collapse more in a short cylinder than in a longer one of similar diameter. For this reason, the resistance to collapse is a function of the length as well as of the diameter and thickness of walls. Unwin shows (page 83) the characteristic forms of collapsed flues for given ratios of length to diameter; and he has deduced a theoretical formula, eq. (23), based upon Euler's formula for long columns. The condition of a circumferential strip of the cylinder under external pressure is similar to that of a long column, and just as Euler's formula ordinarily gives too high a value for the strength of a column, eq. (23) of Unwin is found not to accord very closely with observed results on the resistance to collapse of flues

31. **Collapse of Boiler Flues; Collapse Rings**.—[Unwin, §§ 41–42, pages 84–88.] Fairbairn derived an empirical formula from his experiments on collapsing tests of flues, which is given by Unwin, eq. (24), page 84. This indicates that the collapsing

pressure does not increase as rapidly as the cube, but only a little more rapidly than the square of the thickness. The more convenient approximate expression

$$p = C \frac{t^2}{l d}$$

is frequently used in connection with computations of boiler flues. Fairbairn's value of the coefficient C, is 9,672,000 for *collapsing* pressure, and Unwin gives it as 3,500,000 for *working* pressure, [see eq. (b), page 88]. This last value is obtained from examination of actual flues "30 feet length and 30 to 36 inches in diameter," probably of the Lancashire type of boiler; but the factor of safety (rather less than 3), is certainly low. The Lloyds regulation (British) allows the following pressure in boiler flues and furnaces

$$p = \frac{1{,}075{,}200\, t^2}{l d} \tag{1}$$

$$\therefore\ t = \frac{\sqrt{l d p}}{1037} \tag{2}$$

all dimensions in inches. and pressure in pounds per square inch. The U. S. Board of Supervising Inspectors of Steam Vessels (U. S. B. S. I.) has adopted these same regulations.

The Rules also provide that, in flues reinforced by collapse rings of specified dimensions, the length between such rings is to be taken as the length of the flue in the above formulas. [Consult Rules U. S. B. S. I. of Steam Vessels, § 433; also Unwin § 42, pages 85–86].

If a cylinder under external pressure could be depended upon to fail only by actual crushing, instead of through collapse (buckling) the formula

$$p = \frac{2 f t}{d} \tag{3}$$

would apply, as in internal pressure [see eq. (2), art. 27]; remembering that the stress (f) is compression under external pressure. If eq. (3) gives a lower working pressure than eq. (1), the flue

designed by eq. (3) will be safe against collapse, (see Unwin, page 88). If f be taken at 4,000 (a value allowed by the British Board of Trade and rather less than that allowed for lap welded or riveted flues by the U. S. B. S. I.) equation (3) may be used when $l < 134.4$, for when

$$p = \frac{2ft}{d} = \frac{8{,}000\,t}{d} < \frac{1{,}075{,}200\,t^2}{ld}$$

$$\therefore\ l < 134.4\,t.$$

32. **Corrugated Furnace Flues.**—Flues corrugated, as in Fig. 33, are very much stiffer against collapse than plain cylindrical flues, and may be safely made of any desired length, with proper dimensions of corrugations. When the corrugations are not less than 1½ inches deep, not more than 8 inches centre to centre of corrugations, and plain portions at the ends do not exceed 9 inches, the U. S. B. S. I. allows a working pressure of

$$p = \frac{14{,}000\,t}{d} \tag{1}$$

This is also the formula used by the British Board of Trade.

V.

RIVETED JOINTS.

33. General Considerations of Riveted Joints —[Unwin, §§ 47–48, pages 95–102.]

34. Size of Rivets for Plates of different Thicknesses.—[Unwin, § 49, pages 102–103.]

In punching plates which are quite thin, relative to the diameter of the punch, the action approaches pure shearing, and the relation given by Unwin on page 102 is correspondingly exact. If, however, the thickness is so great that the pressure between the end of the punch and the plate reaches the intensity at which the metal of the plate will flow, the hole may be formed by a combined lateral flow and a shear. Time is required for the change in molecular arrangement which occurs during the flow of a ductile metal; but if the motion of the punch is not too rapid, good, ductile wrought iron, or soft steel, will flow under the punch, before the crushing stress of a properly tempered tool is reached. It is thus possible to force a punch through a plate of such material when the thickness of plate is several times the diameter of the punch. Hoopes & Townsend, of Philadelphia, punched holes $\frac{7}{16}''$ diameter in wrought iron $1\frac{3}{4}''$ thick, and it is stated that a single punch made 585 of such holes. In this case the pressure on the end of the punch would have been about 650,000 lbs. per square inch, had the metal been simply sheared; while it is probable that flow began under a pressure of about one-tenth this intensity.

The lateral flow of the metal in this instance was evidenced by the fact that the "wad," or punching, from one of these holes did not contain one-half as much metal as was displaced in making the hole.

The pressure of flow of ductile wrought iron and mild steel is

probably not ordinarily over 60,000 to 70,000 lbs. per square inch, which is well within the crushing resistance for tempered tool steel, in the absence of severe shock.

The injury to plates by punching, to which Unwin refers on page 96, is due to this lateral flow; and it becomes more important with an increase in thickness of the plates.

35. Lap of Plates and Pitch of Rivets.—[Unwin, § 50, page 103.]

36. Forms of Riveted Joints.—[Unwin, § 51, pages 104–107.] The figures in this section show several standard forms of riveted joints. Numerous other forms are in use; but the methods of computation to be given in art. 38 may readily be extended to cover any case.

37. Modes of fracture of Riveted Joints.—[Unwin, §§ 52–53, pages 107–108.] Of the four methods of fracture mentioned by Unwin, the third (breaking out of the plate in front of the rivet) may always be avoided by giving sufficient lap. Hence eq. (4) of § 53 (Unwin) may be neglected.

The only objection, in practice, to large lap is the somewhat greater difficulty in securing a tight joint by caulking.

The resistance to the tearing of the plates, shearing of rivets, or crushing of the rivets (or plates) are inter-dependent, and for given materials of plates and rivets there are definite relations between the pitch and diameters of rivets, for any given form of joint, which cannot be departed from without sacrifice of strength in the joint as a whole. This will appear from the discussion in the next article.

38. Strength of Riveted Joints.—Any riveted joint may be considered as consisting of a number of unit strips, all alike as to width, number of rivets, pitch, etc. Thus, in a single riveted joint each strip has a width equal to the pitch of the rivets, and contains one rivet, (see Unwin, Figs. 48 to 51, page 107.) With the ordinary double riveted joint, the unit strip contains two rivets and its width is equal to the pitch of rivets in either row. Figs. 44 and 45 (Unwin) show portions of double riveted joints each roughly equal to two such unit strips. A similar division of

the joint into unit strips, whatever its form, is evidently possible. The computations for strength against tearing of the plates, shearing of rivets, etc., may be made for such a unit strip, as every other equal strip would have the same computed strength.

The following notation will be used throughout this article:

d = diameter of hole, (diam. of rivet when upset.)

p = pitch of rivets along a row.

t = thickness of the plates.

f_t = tensile strength of the plates per sq. in.

f_c = crushing strength of the rivets, or plates, per sq. in.

f_s = shearing strength of rivets in single shear per sq. in.*

f_s' = shearing strength of rivets in double shear per sq. in.

C = Crushing resistance of rivets, or plates, per unit strip.

S = Shearing resistance of rivets per unit strip.

T = Tensile resistance of plates (net section) per unit strip.

P = Tensile resistance of plates (solid section) per unit strip.

E = Efficiency of joint.

The following clearly indicates the meaning of these symbols:

I. SINGLE RIVETED LAP JOINTS.

[See Unwin, Figs. 48 to 51; also equations (2), (3) and (5), § 53.]

The net tensile strength of the strip (through the rivet) is

$$T = (p - d)\,t f_t. \qquad (1)$$

The shearing strength of the rivet is

$$S = \frac{\pi}{4} d^2 f_s = .7854\, d^2 f_s. \qquad (2)$$

The crushing strength of rivet, or plate, whichever has the lower crushing resistance, is

$$C = d t f_c. \qquad (3)$$

---

*Experiments show that rivets in double shear do not usually have as great strength (per square inch sheared) as similar rivets in single shear. Or the resistance of the two sections in double shear is not twice that of the one section in single shear. On the other hand, the crushing resistance is probably higher in double shear rivets, because of their more uniform bearing. This is neglected in the present article.

The tensile strength of the solid plate; that is, of a section across the strip not passing through a rivet hole, is, per unit strip

$$P = p\,t\,f_t. \qquad (4)$$

The *efficiency of the joint*, E, is the ratio of the strength of the joint to the strength of the solid plate; or it is the smallest of T, S, or C, divided by P. For highest efficiency, T, S and C should be equal. If the proportions are such that this result is attained, the three equations (1), (2) and (3) involve the three unknown quantities: p, d, and $T = S = C$; the terms t, f_t, f_s and f_c being known. Equating S and C, eqs. (2) and (3).

$$\frac{\pi}{4} d^2 f_s = d t f_c \quad \therefore\ d = 1.27 t \frac{f_c}{f_s} \qquad (5)$$

This expression gives the proper theoretical diameter of rivets for plates of a given thickness, when the values of f_s and f_c are fixed. Equating T and S, eqs. (1) and (2)

$$(p - d)\,t f_t = \frac{\pi}{4} d^2 f_s \quad \therefore\ p = \frac{.7854\, d^2 f_s}{t f_t} + d \qquad (6)$$

This gives the proper theoretical pitch.

Equation (5) generally gives a diameter which does not exactly agree with regular "shop dimensions," hence the diameter of actual rivets (holes) would be varied to give a convenient size. Likewise, the pitch would perhaps be made somewhat different from that computed, for convenience in laying off, or to make even spacing in the entire length of the plate. These changes will destroy the exact equality between T, S and C. The substitution of the final practical values of d and p in equations (1), (2), (3) and (4) will give the values of T, S, C and P, and the smallest of the first three of these values divided by the value of P gives the efficiency, E.

Example: A single riveted lap joint is to be designed for a plate $\frac{1}{2}$ inch thick. Tensile strength of plate = 58,000 pounds per square inch; shearing strength of rivets (single shear) = 40,000; and crushing resistance taken at 70,000.

$$d = 1.27 \times .5 \times 70{,}000 \div 40{,}000 = 1.11 \text{ inches.}$$

The rivet (hole) will be taken as $1\frac{1}{16}$ inch diameter (1 inch rivets). Using this value of d in eq. (6)

$$p = \frac{.7854 \times 289 \times 40{,}000}{.5 \times 256 \times 58{,}000} + \frac{17}{16} = 1.22 + 1.0625 = 2.28''$$

The pitch may be taken at $2\frac{5}{16}''$, unless some slightly different pitch would space to better advantage.*

From eq. (1)

$$T = (2.31 - 1.06) \times .5 \times 58{,}000 = 36{,}300 \text{ lbs.}$$

From eq. (2)

$$S = .7854 \times 1.13 \times 40{,}000 = 35{,}400 \text{ lbs.}$$

From eq. (3)

$$C = 1.0625 \times .5 \times 70{,}000 = 37{,}200 \text{ lbs.}$$

From eq. (4)

$$P = 2.31 \times .5 \times 58{,}000 = 67{,}000 \text{ lbs.}$$

∴ The efficiency of the joint equals tbe smallest of these resistances, S in this case, divided by the strength of the solid plate, P.

$$\therefore E = S \div P = 35{,}400 \div 67{,}000 = .53,$$

or the efficiency is 53 per cent.

II. DOUBLE RIVETED LAP JOINTS.

The unit strip contains two rivets, one in each row; but its cross-section through either rivet is only weakened by a single hole, hence

$$T = (p - d)\,t f_t \qquad (7)$$

The shearing resistance of the two rivets in this unit strip is

$$S = \frac{2\pi}{4} d^2 f_s = 1.57\, d^2 f_s \qquad (8)$$

The crushing resistance of the two rivets is

$$C = 2\, d\, t\, f_c \qquad (9)$$

The tensile strength of the solid plate per unit strip is

$$P = p\, t f_t \qquad (10)$$

*It is preferable to slightly increase, rather than to decrease the pitch as computed; because corrosion weakens the plates more than it does the rivets.

Equating S and C, eqs. (8) and (9)

$$1.57\,d^2 f_s = 2\,d\,t\,f_c \quad \therefore \quad d = 1.27\,t\,\frac{f_c}{f_s} \tag{11}$$

which gives the same diameter of rivet, for the same thickness of plates and materials, as in the case of a single riveted lap joint.

Equating T and S, eqs. (7) and (8),

$$(p-d)\,t\,f_t = 1.57\,d^2 f_s \quad \therefore \quad p = \frac{1.57\,d^2 f_s}{t\,f_t} + d \tag{12}$$

The process in designing this joint is similar to that given for single riveted joints. First, the diameter of rivet is computed from eq. (11), and some regular size of nearly this diameter is adopted. Second, the pitch is computed from eq. (12), and this may be modified to give a convenient dimension. Then T, S, and C are computed from eqs. (7), (8) and (9), respectively, and the smallest of these divided by P,—as found from eq. (10),—gives the efficiency.

III. TRIPLE RIVETED LAP JOINTS.

If a triple riveted lap joint is made with three rows of rivets (all rows having the same pitch), there would be three rivets in each unit strip of width equal to the common pitch. The equations would then be

$$T = (p-d)\,t\,f_t \tag{13}$$

$$S = \frac{3\pi d^2}{4} f_s = 2.356\,d^2 f_s \tag{14}$$

$$C = 3\,d\,t\,f_c \tag{15}$$

$$P = p\,t\,f_t \tag{16}$$

$$d = 1.27\,t\,\frac{f_c}{f_s} \tag{17}$$

$$p = \frac{3\pi d^2 f_s + 4\,d\,t\,f_t}{4\,t\,f_t} = \frac{2.356\,d^2 f_s}{t\,f_t} + d \tag{18}$$

And the design would be carried out as in the preceding cases.

It will be shown, however, that a higher efficiency is obtained

by giving the joint the form shown in Fig. 46 (Unwin), in which the two outer rows have twice the pitch of the inner rows. In this second form of the triple riveted joint, there are four rivets per unit strip ; the width of this strip being $p' = 2p$, in which p is the pitch of the middle row. For such a unit strip

$$T' = (p' - d)\, t f_t \qquad (13')$$

$$S' = \frac{4\pi d^2}{4} f_s = \pi d^2 f_s \qquad (14')$$

$$C' = 4\, d\, t f_c. \qquad (15')$$

$$P' = p' t f_t = 2\, p\, t f_t. \qquad (16')$$

Equating S' and C', eqs. (14′) and (15′)

$$\pi d^2 f_s = 4\, d\, t f_c \quad \therefore \quad d = 1.27\, t \frac{f_c}{f_s}. \qquad (17')$$

Equating T' and S', eqs. (13′) and (14′)

$$p' = \frac{\pi d^2 f_s + d\, t f_t}{t f_t}. \qquad (18')$$

These equations are used as in the preceding cases. First, finding d from eq. (17′) ; then finding p' from eq. (18′) ; finally computing T', S', C' and P', and getting the efficiency by dividing the smallest of the first three resistances by P'.

To show that this modified triple riveted joint (Fig. 46, Unwin) is more efficient than three rows of rivets with equal pitch, the general expressions for efficiency of each form will be derived. It will be assumed that each joint is of maximum strength for its form ; that is, that $T = S = C$, and that $T' = S' = C'$. The efficiency would thus be given for the first form by dividing T, S, or C by P; and for the second case by dividing T', S', or C' by P'.

For the first form of joint, from eqs. (14), (16) and (18)

$$E = S \div P = \tfrac{3}{4}\pi d^2 f_s \div p\, t f_t = \tfrac{3}{4}\pi d^2 f_s \div \left[\frac{3\pi d^2 f_s + 4\, d\, t f_t}{4\, t f_t}\right] t f_t.$$

$$= \frac{\pi d^2 f_s}{\pi d^2 f_s + \frac{4}{3}\, d\, t f_t} \qquad \text{(a)}$$

For the second form of joint, from eqs. (14′), (16′) and (18′)

$$E' = S' \div P' = \pi d^2 f_s \div p' t f_t = \pi d^2 f_s \div \left[\frac{\pi d^2 f_s + d t f_t}{t f_t}\right] t f_t = \frac{\pi d^2 f_s}{\pi d^2 f_s + d t f_t}. \qquad \text{(a}'\text{)}$$

Since the only difference in the two equations (a) and (a′) is that the latter has the smaller denominator, E' is greater than E; or the joint with outer rows having twice the pitch of the inner rows is of higher efficiency than the form with the same pitch in all rows, provided each joint is designed for its maximum efficiency.

Butt joints with a single covering strip, or welt, (Unwin, Figs. 43 and 45) are sometimes used for circumferential seams of a boiler shell ; or, (with countersunk rivets, Unwin, Fig. 39), in ship work, where a smooth exterior surface is desired. Single welt butt joints would be designed by the formulas for lap joints with the corresponding number of rows of rivets ; for the covering strip forms a lap joint with each of the adjacent plates.

IV. SINGLE RIVETED BUTT JOINTS, TWO WELTS.

This joint is not very commonly used, unless in the circumferential seams of boiler shells which have double riveted butt joints with two welts for the longitudinal seams. As the circumferential seams require only half the strength of the longitudinal seams (see art. 27), it is the common practice to make the circumferential joints of a form having lower efficiency than the longitudinal joints. Even single riveted lap joints have an efficiency greater than 50 per cent. (with any ordinary materials) ; hence they might be safely used for circumferential seams with any possible efficiency in the longitudinal joints. Convenience often dictates a butt joint for circumferential seams, as this reduces the difficulty of disposing of the welts of the other joints. (consult Unwin, Fig. 58.)

In joints with two welts, each somewhat thicker than one-half the main plates, the bearing area of the rivets against the plates

(which determines the resistance to crushing) is dt, as in lap joints; but each rivet presents two cross-sections to be sheared. As stated in the footnote on page 74, the resistance of a rivet to double shear cannot be assumed at twice the resistance, per unit of area, of the same rivet in single shear. The notation at the head of this article gives f_s' as the symbol for unit shearing stress in double shear.

The single riveted butt joint with two welts is similar in form to that shown by Unwin, Fig. 47, except that there is only one row of rivets each side of the butt, and the welt is correspondingly narrower. In such a single riveted butt joint, the unit strip has a width p, and it contains one rivet (each side of the butt) which is in double shear. The equations are as follows:

$$T = (p - d)tf_t \tag{19}$$

$$S = \frac{2\pi d^2 f_s'}{4} = 1.57\, d^2 f_s' \tag{20}$$

$$C = d t f_c \tag{21}$$

$$P = p t f_t \tag{22}$$

Equating S and C, eqs. (20) and (21)

$$\frac{\pi d^2 f_s'}{2} = d t f_c \quad \therefore \quad d = .636\, t \frac{f_c}{f_s'} \tag{23}$$

which gives the proper diameter of rivets, to be modified, as before, for convenient shop dimensions.

Equating T and S, eqs. (19) and (20)

$$(p - d)tf_t = \frac{\pi}{2} d^2 f_s' \quad \therefore \; p = \frac{1.57\, d^2 f_s'}{t f_t} + d \tag{24}$$

Taking the nearest convenient pitch to that computed by eq. (24), the values of T, S, C and P are found as in the preceding cases, and the smallest of the first three divided by P gives the efficiency.

V. DOUBLE RIVETED BUTT JOINTS, TWO WELTS.

In this joint, if the rivets in each row have the same pitch, the unit strip has a width equal to the pitch; each unit strip contains two rivets in double shear, and the equations are

$$T = (p - d)\, t f_t \qquad (25)$$

$$S = \frac{2 \times 2\pi}{4} d^2 f_s' = \pi d^2 f_s' \qquad (26)$$

$$C = 2\, d\, t f_c \qquad (27)$$

$$P = p\, t f_t \qquad (28)$$

Equating S and C, eqs. (26) and (27)

$$d = \frac{2\, t f_c}{\pi f_s'} = .636\, t \frac{f_c}{f_s'} \qquad (29)$$

Equating S and T, eqs. (25) and (26)

$$p = \frac{\pi d^2 f_s'}{t f_t} + d \qquad (30)$$

If the double riveted butt joint be made like that shown by Fig. 47 (Unwin) with the outer rows twice the pitch of the inner rows, the efficiency of the joint may be increased, as in the similarly modified triple riveted lap joint. This modified double riveted butt joint has a unit strip of width $p' = 2p$, p being the pitch of the rivets in the inner rows. Each unit strip contains three rivets in double shear. The equations for this joint are

$$T' = (p' - d)\, t f_t \qquad (25')$$

$$S' = \frac{6\pi d^2 f_s'}{4} = 4.72\, d^2 f_s' \qquad (26')$$

$$C' = 3\, d\, t f_c \qquad (27')$$

$$P' = p'\, t f_t \qquad (28')$$

Equating S' and C', eqs. (26') and (27')

$$d = .636\, t \frac{f_c}{f_s} \qquad (29')$$

Equating T' and S', eqs. (25') and (26')

$$p' = 2p = \frac{3\pi d^2 f_s}{2\, t f_t} + d \qquad (30')$$

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VI. TRIPLE RIVETED BUTT JOINTS, TWO WELTS.

The joint to be considered is the modified form shown by Fig. 34. It consists of two rows of rivets in double shear and one outer row in single shear, each side of the butt, the pitch of rivets in the outer rows being twice that of the inner rows. One of the covering strips is only wide enough to take the two close pitch rows, while the other strip is wide enough for the three rows each side of the butt. This form of joint has a high efficiency, and is much in favor for the best class of work under heavy pressure. The unit strip contains four rivets in double shear and one in single shear, and its width equals the pitch of outer row, $p' = 2p$.

The equations are

$$T = (p' - d)\,t f_t. \qquad (31)$$

$$S = \frac{2 \times 4 \pi d^2 f_s'}{4} + \frac{\pi d^2 f_s}{4} = \pi d^2 \left(2 f_s' + \tfrac{1}{4} f_s \right) \qquad (32)$$

$$C = 5\,d\,t f_c. \qquad (33)$$

$$P = p'\,t f_t. \qquad (34)$$

Equating S and C, eqs. (32) and (33)

$$d = \frac{5\,t f_c}{\pi(2 f_s' + \frac{1}{4} f_s)} = \frac{1.59\,t f_c}{2 f_s' + \frac{1}{4} f_s} = \frac{6.36\,t f_c}{8 f_s' + f_s} \qquad (35)$$

Equating T and S, eqs. (31) and (32)

$$p' = 2p = \frac{2 \pi d^2 f_s' + \frac{1}{4} \pi d^2 f_s}{t f_t} + d = \left(\frac{6.28 f_s' + .785 f_s}{t f_t} \right) d^2 + d. \qquad (36)$$

There are many other possible arrangements of rivets in a joint, but the preceding include most of the usual forms, and the derivation of the above equations will suggest the method of finding those for any given style of joint.

39. **General Equations for Riveted Joints.**—The fundamental equations for efficiency of riveted joints of various styles, as well as those for the diameter and pitch of the rivets, may be

put in more general forms than those of the preceding article. The equations developed in the present article are applicable to any style of riveted joint.

The unit strip is of width equal to the pitch ; the maximum pitch being taken for such width of unit strip if all rows do not have the same pitch, as in the modified double and triple riveted joints.

The general expression for the net tensile strength of the unit strip is

$$T = (p - d) t f_t. \qquad (1)$$

The general expression for resistance to shearing of the rivets in the unit strip is

$$S = \frac{n \pi d^2}{4} f_s + \frac{2 m \pi d^2}{4} f_s' \qquad (2)$$

in which n equals the number of rivets in single shear and m equals the number of rivets in double shear.

The general expression for resistance to crushing of the unit strip is

$$C = n d t f_c + m d t f_c'. \qquad (3)$$

This is a recognition of the fact that the resistance to crushing with single shear (f_c) is probably less than the resistance with double shear (f_c'), owing to the more uniform distribution of bearing pressure in the latter case ; such a distinction was not made in the equations of the preceding article.

The tensile resistance of the solid strip is

$$P = p t f_t. \qquad (4)$$

Equating S and C, eqs. (2) and (3)

$$\frac{\pi d^2}{4} (n f_s + 2 m f'_s) = d t (n f_c + m f_c')$$

$$\therefore d t = \frac{\pi d^2 (n f_s + 2 m f_s')}{(n f_c + m f_c')} \qquad (5)$$

Equating T and S, eqs. (1) and (2) and solving

$$p t f_t = \frac{\pi d^2}{4} (n f_s + 2 m f_s') + d t f_t = P. \qquad (6)$$

If the joint is designed for maximum efficiency, $T = S = C$, hence any one of these three quantities divided by P gives the efficiency of the ideal joint, for any given form, or dividing (2) by (6)

$$E = \frac{S}{P} = \frac{\frac{\pi d^2}{4}(n f_s + 2 m f_s')}{\frac{\pi d^2}{4}(n f_s + 2 m f_s') + d t f_t}$$

Substituting the value of $d t$ as given by eq. (5) and dividing numerator and denominator by $\frac{\pi d^2}{4}(n f_s + 2 m f_s')$,

$$E = \frac{1}{1 + \frac{f_t}{n f_c + m f_c'}} \qquad (7)$$

If all the rivets are in single shear

$$E = \frac{1}{1 + \frac{f_t}{n f_c}}. \qquad (7')$$

If all the rivets are in double shear

$$E = \frac{1}{1 + \frac{f_t}{m f_c'}}. \qquad (7'')$$

Or if $f_c = f_c'$, (as assumed in art. 38), and the number of rivets in the unit strip, $n + m$, be called K,

$$E = \frac{1}{1 + \frac{f_t}{K f_c}}. \qquad (7''')$$

This holds for any form of the joint. The formula (7), or the appropriate modified form of it (7′), (7″), or (7‴), may be used to find the maximum possible efficiency of any *form* of riveted joint when the resistances to tension and crushing are known. It will be noticed that the resistance of the rivets to shearing does not appear in this general formula for maximum efficiency. It is thus

seen that, with given resistance to tension and crushing, the shearing strength of the rivets does not affect the attainable efficiency. However, the shearing resistance of the rivets does affect the proportions of the joint necessary for such maximum efficiency.

This formula is useful in finding the limiting efficiency of joint for any form and materials; the actual proportions adopted may give a lower efficiency, but can never give higher efficiency.

It is possible to derive a set of general expressions for the diameter of rivets and for their pitch which can be used for any form of joint. The notation used is the same as in the preceding work of this article.

Equating S and C, eqs. (2) and (3) and solving for d

$$d = \frac{4}{\pi} \times \frac{n f_c + m f_c'}{n f_s + 2 m f_s'} t \qquad (8)$$

which gives the proper diameter of rivets for a given thickness of plate, when the number of rivets in single shear and the number in double shear and the corresponding shearing and crushing resistance are known.

Equating T and C, eqs. (1) and (3), and solving for p

$$p = \frac{(n f_c + m f_c')}{f_t} d + d \qquad (9)$$

Or, equating T and S, eqs. (1) and (2), and solving for p

$$p = \frac{\pi}{4} d^2 \left(\frac{n f_s + 2 m f_s'}{t f_t} \right) + d \qquad (10)$$

In case of simple lap joints, all rivets are in single shear, hence

$$d = \frac{4 f_c}{\pi f_s} t \qquad (8')$$

$$p = \frac{n f_c}{f_t} d + d \qquad (9')$$

Or $$p = \frac{\pi}{4} d^2 \frac{n f_s}{t f_t} + d \qquad (10')$$

In case of simple butt joints with two welts through which all the rivets pass, the rivets are all in double shear, hence

$$d = \frac{2 f_c'}{\pi f_s'} t \tag{8''}$$

$$p = \frac{m f_c'}{f_t} d + d \tag{9''}$$

Or $$p = \frac{\pi}{2} \frac{m f_s'}{t f_t} d^2 + d \tag{10''}$$

If, in the general form of joint, f_s' is taken as equal to f_s and $f_c' = f_c$

$$d = \frac{4}{\pi} \frac{(n + m) f_c}{(n + 2 m) f_s} t \tag{8'''}$$

$$p = \frac{(n + m) f_c}{f_t} d + d \tag{9'''}$$

Or $$p = \frac{\pi}{4} d^2 \frac{(n + 2 m) f_s}{t f_t} + d \tag{10'''}$$

These general equations are due to Mr. William N. Barnard, who also suggested the above expressions for the maximum efficiency in the general case.

40. Customary Proportions of Riveted Joints.—The method of computation indicated in Article 38 should be used when the necessary data is available, especially for joints subjected to high pressure. It is apparent that any variation in the strength of the material used would affect the proportions when such methods are employed in designing a joint for maximum efficiency, and that a table of rivet diameters and pitches would have to be very extensive to cover the entire range of practice. However, it is not uncommon to adopt regular diameters and pitches for a given thickness of plate and form of joint; of course such proportions would only give the best efficiency for certain combinations of shearing, crushing and tensile strengths. The following formulas are derived from Tables given by the Hartford Steam Boiler Inspection and Insurance Co. for lap joints, using the following notation :

$d =$ diameter of hole $=$ diam. of rivet $+ \frac{1}{16}$ inch.
$p =$ pitch of rivets.
$t =$ thickness of plates.
All dimensions in inches. Iron rivets.

For Iron Plates, $d = t + \frac{7}{16}''$. (1)
For Steel Plates, $d = t + \frac{1}{2}''$. (2)
Single Riveted Lap Joints $p = t + 1\frac{3}{4}''$. (3)
Double Riveted Lap Joints, $p = 2\,t + 2\frac{1}{2}''$. (4)

Various hand-books and treatises on boiler construction give tables of this character; but it is well to check such values before adopting them for other than light service.

41. Strength of Iron and Steel used for Boiler Plates and Rivets. [Unwin, § 54, page 109.] Ductility is of even greater importance than strength in boiler materials, as the straining actions due to pressure, unequal expansion, etc., are often distributed very unequally. Hence it is important to use materials which will distribute the stress, through yielding, before local rupture occurs. For these reasons only soft wrought iron and steel are used in this class of work. The tensile strength of the usual grades of boiler plates may be assumed at about the values given by Unwin in § 54.

The shearing strength of wrought iron rivets has been given by Mr. J. M. Allen, President of the Hartford Steam Boiler Insurance and Inspection Co., at 38.000 lbs. per square inch for single shear, and at 35,000 lbs. per square inch for double shear. The values are based on tests made at the Watertown Arsenal. Other authorities assume somewhat higher values. Wrought iron rivets have remained in great favor even with the general adoption of steel plates; but steel rivets are now being largely used. Steel rivets may be assumed to have a strength of 45.000 lbs. per square inch in single shear, and perhaps 40.000 to 42,000 lbs. per square inch in double shear. The crushing resistance of rivets and plates is not quite so defiinitely known, but it may be taken at from 60,000 to 70,000 lbs. per square inch.

42. Apparent Strength of Perforated Plates. [Unwin, §§

54–55; pages 109–112.] As shown by Unwin (in the discussion of "*Tenacity of drilled plates*", and "*Tenacity of punchea plates*"), the drilled plates generally exhibit a higher apparent unit strength than ordinary test bars of the same material. This is because the drilled plate is equivalent to a row of short specimens placed side by side, and short test bars usually show an excess over the true strength of the material, especially with ductile metals. A punched plate has this same advantage of form; but the injury to the plate by lateral flow in punching often more than compensates for the gain in apparent strength due to the test of "short specimens". If the punched plate is annealed or reamed out after punching, it may show an increase in apparent strength corresponding approximately to that observed in drilled plates. In the computations of riveted joints, it is usual to neglect this element of additional strength.

43. Friction and Binding of Riveted Joints—[Unwin, § 57, pages 112–113]. The friction between the plates due to the tension in the rivets, produced by their contraction in cooling, tends to increase the apparent shearing strength of the rivets. This very uncertain element cannot safely be depended upon, because a slight yielding of the plates and rivets, even under the ordinary service load, may soon greatly reduce the normal pressure between the contact surfaces of the plates. On the other hand, the bending of the plates, in lap or single welt butt joints, may be an important source of weakness. See Unwin, Figs. 42 and 43. Repetition of this bending action may cause a plate to break through the solid section, near the edge of the overlapping plate; and this is especially apt to occur if the plate has been scored by careless use of a caulking tool with a sharp corner. The round nose caulking tool is safer as well as more effective.

44. Distance between Rows of Rivets.—In multiple riveted joints the distance between the rows of rivets should be sufficient to insure a greater strength against tearing of the plate along a zig-zag section than straight through the rivets of a single row. The net zig zag section should be at least 1⅓ times the straight section; when the diagonal pitch (distance from centre of a rivet

in one row to the centre of the nearest rivet in the adjacent row) is given by the following expression. If p_1 = the diagonal pitch, p the straight pitch of the inner rows, and d the diameters of the rivets,

$$p_1 = \tfrac{2}{3} p + \tfrac{1}{3} d. \qquad (1)$$

Unwin gives as the minimum diagonal pitch, twice the diameter of the rivets, (§ 50, Fig. 41); while the proportions shown in Fig. 47 (Unwin) allow a distance of $2\,d$ between centres of the parallel rows of rivets. In triple riveted butt joints the outer rows are often placed somewhat farther from the middle rows than the latter are from the inside rows, especially when the outer rows have only half as many rivets (twice the pitch) as the other rows.

45. Graphic Method of designing Joints.—[Unwin, § 64, pages 118–121].

46. Junction of three or more Plates.—[Unwin, § 69, pages 124–126].

47. Connection of Plates not in one Plane.—[Unwin, §§ 70–71, pages 126–130].

48. Position of Rivets in Bars.—[Unwin, § 72, pages 130–131].

49. Cylindrical Riveted Structures.—[Unwin, §§ 73–74, pages 131–135].

50. Stayed Flat Surfaces.—[Unwin, § 75, pages 135–137; also § 46, pages 93, 94]. Copper fire boxes and stays are not now extensively used in this country, having been almost completely superseded by iron and steel.

Short stay bolts between parallel plates, as in the "water-leg" of locomotive boilers, are liable to fracture from the relative motion of the connected plates due to unequal expansion. The fracture occurs at the bottom of the thread near one of the plates, usually at the outside plate. The drilled end stay (Unwin, Fig. 79) is an effective means of calling attention to a broken stay bolt, and the practice of drilling the stays is now quite common in the better class of work.

The relations given near the bottom of page 136 (Unwin), are not generally applicable, under authorized rules, for flat stayed surfaces.

Solving eq. (33)—Unwin, page 93—for the pressure,

$$p = \left(\frac{9}{2}f\right)\frac{t^2}{a^2} \tag{1}$$

In which p = working pressure of steam in pounds per sq. inch; f = allowable stress in plate in pounds per sq. inch; t = thickness of the plate in inches, and a = the pitch (distance center to center) of the stay bolts in inches.

If f be taken at 6,000 lbs. per square inch,

$$p = \frac{9 \times 6{,}000}{2}\,\frac{t^2}{a^2} = 27{,}000\,\frac{t^2}{a^2} \tag{2}$$

The Lloyds rule for flat stayed surfaces is

$$p = \frac{C(16t)^2}{a^2}* \tag{3}$$

in which

C = 90 for iron or steel plates $\frac{7}{16}$ inch thick or less, with stays screwed into the plates and riveted over at ends.

C = 100 for iron or steel plates over $\frac{7}{16}$ inch thick, with screwed stays riveted over.

C = 110 for iron or steel plates $\frac{7}{16}$ inch or less, with screwed stays and nuts.

C = 120 for iron plates over $\frac{7}{16}$ inch thick, or for steel plates over $\frac{7}{16}$ inch and less than $\frac{9}{16}$ inch thick, with screwed stays and nuts.

C = 135 for steel plates $\frac{9}{16}$ inch thick or over, with screwed stays and nuts.

C = 140 for iron plates with double nuts (nuts inside and outside of plate).

C = 150 for iron plates with double nuts or stays, and washers on the outside of at least half the thickness of the plates, and of diameter not less than one third the pitch of stays.

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*The form $(16t)^2$ in eq. (3) is convenient for computations, for it equals the square of the number of sixteenths of an inch in the thickness of the plate.

$C = 160$ for iron plates, with double nuts or stays, and washers riveted to outside of plates, washers having a diameter not less than two-fifths the pitch of stays and thickness of at least one-half the thickness of the plates.

$C = 175$ for iron plates, with double nuts or stays, and washers riveted to outside of plates, washers having diameter at least two-thirds the pitch of stays and thickness equal to that of the plates.

For steel plates, except for combustion chambers directly exposed to the heated gases, C may be increased from 140 to 175, from 150 to 185, from 160 to 200, and from 175 to 220, in the above cases.*

It will be noticed that the Lloyds formula, eq. (3), is of the same form as that given in eqs. (1) and (2). The constant of eq. (2) is nearly equivalent to 105 in eq. (3).

Unwin gives the rule of the Board of Trade (British) with the values of the constants, on pages 93 and 94. This rule is preferred by some authorities; but it is not quite so simple in form as the Lloyds rule and as the latter is unquestionably safe, it will be used in this work.

The stay bolt itself should have a minimum area of cross-section (at the bottom of the threads) sufficient to carry a load $= pa^2$, with a stress not over 6,000 to 7,000 for wrought iron, and 7,000 to 8,000 for steel.

51. **Diagonal Stays.**—[Unwin, § 76, pages 137–138].

Gusset stays (Unwin, Fig. 81) are liable to unequal tension across the thin broad section, so that the maximum intensity of stress at the edge may be excessive, even though the nominal (mean) intensity of stress is quite moderate. The diagonal stay (Unwin, Fig. 80) is to be preferred to the gusset stay.

52. **Bridge or Girder Staying.**—[Unwin, § 76a, pages 138–141].

53. **Direct Stays.**—In marine boilers, or others having large diameter and relatively small length, the portions of the flat

* These values are taken from Low's Pocket Book for Mechanical Engineers.

heads which are above the tubes are commonly stayed by long bolts passing through the steam space from one head to the other. These bolts have nuts with washers on the outside, and they should be spaced so that each of the different bolts support approximately the same area of plate. The smallest diameter of the stay is given by the equation

$$d = \sqrt{\frac{p\,A}{\frac{1}{4}\pi f}} = 1.127\sqrt{\frac{p\,A}{f}} \qquad (1)$$

in which p is the steam pressure, A the area of plate supported by the stay, and f the allowable intensity of stress in the stay. The Board of Trade allows the following values of stress, $f = 5{,}000$ pounds per sq. inch for welded wrought iron stays; $f = 7{,}000$ for stays of wrought iron from solid bars; $f = 9{,}000$ for steel stays which have not been welded or worked after heating.

The greater portion of the heads below the water line is stayed by the tubes, in tubular forms of boilers. These tubes are usually expanded to closely fill the holes bored in the tube sheets or heads; then the projecting ends of the tubes are "beaded", or riveted over. The tensile stress in such tubes acting as stays may be taken at about 6,000 pounds per square inch of cross-section of metal for wrought iron, and about 7,000 to 7,500 for steel.

VI.

SCREWS, BOLTS, AND KEYS.

54. **Ordinary uses and forms of Screws.**—[Unwin, §§ 77, 78, pages 142 to 146.]

The triangular section threads are best for fastenings, as in ordinary screws, studs, and bolts. This form of thread has more friction between the threads of the screw and the nut than corresponding square threads, thus reducing the liability to unscrew. The resistance to stripping of the threads is also greater than in "square threads," for similar thickness of nuts. On the other hand, the lower frictional resistance of the square thread screw makes this form suitable for transmission of energy. The angular or "V" thread has one advantage over the square thread for such cases as the lead screws of lathes in which lost motion due to wear is seriously objectionable ; because considerable wear of the threads can be taken up by closing the split clamp nut ; while lost motion in the true square thread cannot be taken up in this way. To obtain this great practical advantage without the excessive friction of "V" threads, an intermediate form of thread is frequently used in which the angle between the sides of the threads is 29°, instead of 60° as in the common angular thread. The recognized standard screw thread in the United States is the Sellers, U. S., or Franklin Institute thread. Consult Fig. 86 (Unwin), and the table on page 94 of these Notes. This standard is not used exclusively, however, but a full "V" thread without the flattened tops and bottoms) is in common use. The angle of such "V" threads is almost always 60° in machine bolts ; and the number of threads per inch usually corresponds to those of the Seller's system, but there are many variations in this particular.

55. **Sellers, or United States Screw Threads.**—[Unwin, § 80, pages 146–147.]

The pitch of screw, (the reciprocal of the number of threads

per inch) is the same in both the Whitworth and the Seller's system for all sizes below $1\frac{5}{8}$ inches diameter, except for $\frac{1}{2}$ inch diameter. For this size the Whitworth system gives 12 threads per inch, while the Sellers system gives 13 threads per inch.

SELLERS, U. S., OR FRANKLIN INSTITUTE STANDARD FOR BOLTS.

SCREW THREADS.				NUTS.		BOLT HEADS.	
d=Outside Diameter of Screw.	N=Number of Threads to an Inch.	d_1=Diameter at Root of Thread = Diam. of Hole in Nut.	Area at Bottom Thread.	W= Width of Nut Between Parallel Sides.	T= Thickness of Nut.	w=Width of Head Between Parallel Sides.	t=Thickness of Head.
Inches.	Number.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
$\frac{1}{4}$	20	.185	.027	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{7}{16}$	$\frac{3}{16}$
$\frac{5}{16}$	18	.240	.045	$\frac{19}{32}$	$\frac{5}{16}$	$\frac{17}{32}$	$\frac{1}{4}$
$\frac{3}{8}$	16	.294	.068	$\frac{11}{16}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{5}{16}$
$\frac{7}{16}$	14	.344	.093	$\frac{25}{32}$	$\frac{7}{16}$	$\frac{23}{32}$	$\frac{3}{8}$
$\frac{1}{2}$	13	.400	.126	$\frac{7}{8}$	$\frac{1}{2}$	$\frac{13}{16}$	$\frac{7}{16}$
$\frac{9}{16}$	12	.454	.162	$\frac{31}{32}$	$\frac{9}{16}$	$\frac{29}{32}$	$\frac{1}{2}$
$\frac{5}{8}$	11	.507	.202	$1\frac{1}{16}$	$\frac{5}{8}$	1	$\frac{2}{16}$
$\frac{3}{4}$	10	.620	.302	$1\frac{1}{4}$	$\frac{3}{4}$	$1\frac{3}{16}$	$\frac{11}{16}$
$\frac{7}{8}$	9	.731	.420	$1\frac{7}{16}$	$\frac{7}{8}$	$1\frac{3}{8}$	$\frac{13}{16}$
1	8	.837	.550	$1\frac{5}{8}$	1	$1\frac{9}{16}$	$\frac{15}{16}$
$1\frac{1}{8}$	7	.940	.692	$1\frac{13}{16}$	$1\frac{1}{8}$	$1\frac{3}{4}$	$1\frac{1}{16}$
$1\frac{1}{4}$	7	1.065	.890	2	$1\frac{1}{4}$	$1\frac{15}{16}$	$1\frac{3}{16}$
$1\frac{3}{8}$	6	1.160	1.028	$2\frac{3}{16}$	$1\frac{3}{8}$	$1\frac{1}{8}$	$1\frac{5}{16}$
$1\frac{1}{2}$	6	1.284	1.293	$2\frac{3}{8}$	$1\frac{1}{2}$	$2\frac{5}{16}$	$1\frac{7}{16}$
$1\frac{5}{8}$	$5\frac{1}{2}$	1.389	1.510	$2\frac{9}{16}$	$1\frac{5}{8}$	$2\frac{1}{2}$	$1\frac{9}{16}$
$1\frac{3}{4}$	5	1.491	1.741	$2\frac{3}{4}$	$1\frac{3}{4}$	$2\frac{11}{16}$	$1\frac{11}{16}$
$1\frac{7}{8}$	5	1.610	2.050	$2\frac{15}{16}$	$1\frac{7}{8}$	$2\frac{7}{8}$	$1\frac{13}{16}$
2	$4\frac{1}{2}$	1.712	2.300	$3\frac{1}{8}$	2	$3\frac{1}{16}$	$1\frac{15}{16}$
$2\frac{1}{8}$	$4\frac{1}{2}$	1.837	2.650	$3\frac{5}{16}$	$2\frac{1}{8}$	$3\frac{1}{4}$	$2\frac{1}{16}$
$2\frac{1}{4}$	$4\frac{1}{2}$	1.962	3.030	$3\frac{1}{2}$	$2\frac{1}{4}$	$3\frac{7}{16}$	$2\frac{3}{16}$

The Sellers screws have much greater tensile strength than full V threads of equal angles and pitch ; because the thread of the former is only three fourths as deep, owing to the flattening at the tops and bottoms. The depth (h) of a full V thread with 60° between the sides is equal to the pitch (p) multiplied by the cosine of 30° ; or .866 p. Hence if d is the outside diameter of the screw and d_1 is the diameter at the bottom of the thread,

$$d_1 = d - 2h = d - 1.732p$$

for the full depth 60° thread. The depth of the Sellers thread is $\frac{3}{4} \times .866\,p = .65\,p$. Hence $d_1 = d - 1.30\,p$.

The area at bottom of a 1″ full 60° thread is .482 square inches; while the area at the bottom of a 1″ Sellers thread is .55 square inches, or 14 per cent. more.

56. **Machine Screws** —The small sizes of screws, with slotted heads, used in metal work are (as in wire and sheet metal) usually designated by numbers rather than by the actual diameter. But in the standard wire gauges, large numbers designate small diameters, while in machine screws large numbers indicate large diameters. The following formula gives the actual outside diameters of machine screws corresponding to the number by which such screws are designated,

$$d = .0131\,N + .057''.$$

This number, N, is simply a designation of the size and is not to be confused with the number of threads per inch (n). The same size of machine screw is often made with several different numbers of threads per inch. These screws are usually specified by naming the size number first, followed by the number of threads per inch. Thus: an 18–20 machine screw means *size* 18, and 20 threads per inch.

57. **Pipe Threads.**—The Briggs system of Pipe Threads is the established standard in the United States. The numbers of threads per inch for the various sizes of pipe are given in article 26, page 62, of these Notes. For fuller detail see Trans. A. S. M. E., Vol. VIII, page 29.

The threads given in § 79 (Unwin) are not followed in the United States.

58. **Straining Action due to Load Applied to Bolts.**—The load applied to bolts is generally one which tends to separate the connected members, and this action is resisted by a tensile stress in the bolts; but bolts are sometimes used to prevent the relative translation of two or more pieces, when a shearing stress is produced in the bolts. When the load force is oblique to the axis, the stress in the bolt may be combined tension and shearing. If

any screw is screwed up under load there is an initial direct stress (tension or compression) and usually a torsional stress due to friction between the threads of the screw and the nut. With bolts or studs screwed up hard, as in making a steam tight joint, the initial tension due to screwing up may be much in excess of that due to the working load. This will be treated more fully later.

If the load applied to the bolt produces a shearing action, the bolt shank should accurately fit the holes in the connected pieces, at least for the portions near the joint; and if S is the load per bolt, d the diameter of the bolt (shank), and f the shearing stress,

$$S = \frac{\pi}{4} d^2 f \therefore f = \frac{4}{\pi} \frac{S}{d^2}$$

In a bolt subjected to a load which produces tension, the minimum cross section sustains the greatest stress. This smallest cross section, in common bolts, is through the bottoms of the threads

If P is the axial load carried on one bolt, f the tensile strength due to such load, and d_1, the diameter of bolt at bottom of threads,

$$P = \frac{\pi}{4} d_1^2 f \therefore f = \frac{4}{\pi} \frac{P}{d_1^2}$$

See Unwin, § 82, page 148.

The value of $\frac{1}{4} \pi d_1$ is given in the Table on page 94, for various sizes of Sellers screws. For a given diameter and pitch of screw, the area at the bottom of threads would be considerably less with full "V" threads.

59. Resilience of Bolts with Impulsive Load.

In bridge work and other cases requiring long bolts, it is very common to make the cross section through the body of the bolt about equal to the section at the bottom of the threads. This may be done by upsetting the ends where the thread is to be cut, or by welding on ends made from stock somewhat larger than that used for the main length of the bolt.

The most apparent result of this practice is to economize material without sacrifice of strength (as the shank still has an area of cross section equal to the threaded portion), and if the weld (when the ends are welded) is perfect, the strength of the

bolt is not reduced. It seems probable that this reason is responsible for the original adoption of this practice, since it has been most generally used in long tie rods. However, in case of bolts liable to shock, there is an even more important reason for such construction ; since it can be shown that the reduced section not only maintains the full strength under static load, but it very greatly increases the capacity of the bolt to resist shock. This last fact has not been very generally recognized, as appears from the common application of such reduced shank bolts only to structures, rather than to machines.

It has been seen that the resistance of a tension member under a static load is determined solely by its weakest section ; while, in a member subjected to shock, impact, or impulsive load, the resistance depends upon the total extent of distortion of the member due to a given intensity of stress.

As shown in art. 8, the maximum stress with impulsive load is

$$p = \frac{W(h+l)}{K l A}$$

For a stress within the E.L.

$$p = \frac{2\,W}{A}\left(\frac{h+l}{l}\right) = \frac{2\,W}{A}\left(\frac{h}{l} + 1\right)$$

This shows clearly that for a given load, W, applied suddenly or with impact, the stress produced in a member of sectional area, A, is greater as l becomes less relative to h. Hence, if l is increased, the stress produced becomes less for a given impulsive action ; or the resistance to such action is greater for a given value of the stress.

If an ordinary bolt is subjected to shock in a direction to produce tension, the stress will be a maximum at the sections through the bottom of the threads ; the bolt will elongate, but the elongation will be confined largely to the very short reduced (threaded) sections, hence the stress will be much less in the larger portion of the bolt. In a Sellers bolt of one inch diameter the area, A, of the shank is .78 sq. inches, while the area, A', at the bottom of threads is only .55 sq. inches. Therefore a

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stress on A' of 30000 lbs. per sq. in. $= \frac{30000 \times .55}{.78} = 21000$ on the full sections. Suppose the elongation per inch of length at a stress of 30000 (taken as the E. L.) is $\frac{1}{1000}''$. Each inch of section A' will elongate $\frac{1}{1000}''$, while each inch of full sectional $A (= .78$ sq. in.) will have a stress of only 21000 lbs., with a corresponding elongation of $\frac{21}{30} \times \frac{1}{1000}'' = .0007''$. Assume the thread to be $1''$ long, and the remainder of the bolt to be $5''$ long. It will appear that the mean stress on the threaded portion ($1''$) is about the mean of 30000 and 21000, or say 25500 lbs. per square inch; as the mean section is an average of .55 and .78 square inches. Hence the elongation for this threaded 1 inch, when the stress on $A' = 30000$, is $.00085''$, while the other $5''$ (of area A) will elongate under this load $5 \times .0007 = .0035''$. The total elongation will then be $l = .00085 + .0035 = .00435$ inches.

$$\text{If } h = \tfrac{1}{10}'',\ W = \frac{A'p}{2} \quad \frac{l}{h+l} =$$

$$\frac{.55 \times 30000}{2} \times \frac{.00435}{.10435} = 8250 \times .0416 = 344 \text{ lbs.}$$

Now suppose the $5''$ shank of this bolt were reduced in section to an area $A' = .55''$. Then the elongation of this portion under the above load would be, $5 \times .001 = .005''$, instead of $.0035''$ and the total elongation would be $l = .00085 + .005 = .00585$.

$$\therefore\ W = \frac{.55 \times 30000}{2} \times \frac{.00585}{.10585} = 8250 \times .0553 = 457 \text{ lbs.}$$

This latter load is 33 per cent. greater than the preceding. With a "V" thread, not reduced in the shank the case would be much worse, as the reduced section is very small in length, theoretically it is zero.

The preceding example shows that the *elastic resilience* of the bolt was increased 33 per cent. by reducing the body of the bolt to A'. Of course the gain would be still greater with a longer bolt. It may be well to remember that the "long specimen" is more apt to contain a weak section than is a short specimen; but, on the other hand, the sharp notching of the threads is quite liable to start a fracture at their roots.

If the bolt is strained beyond the elastic limit, the portion thus strained yields at a much greater rate, relative to the stress, than that given above. With a load which would produce a stress of 30000 lbs. per sq. in. in the larger portion (area A), the stress in the reduced portion (area A') will be $\frac{30000 \times .785}{.55} = 43000$ lbs. per sq. inch. Hence, the effect of a long section in resisting shock *without rupture* is much greater even than that shown for elastic deformation only.

The section of the shank of the bolt may be reduced as in Fig. 35, by turning down the body of the bolt to about the diameter at the bottoms of the threads. The collars a and a' may be left to form a fit in the hole. This form is easy to make, but does not fit the hole throughout its length, and it is weak in torsion.

Fig. 36 is somewhat more expensive, but fits the hole better and is somewhat stronger in torsion. Fig. 37 is the form which gives the best fit, and is also the strongest in torsion. If very long it is difficult to make ; otherwise it is perhaps the best.

These high resilience bolts only increase the resistance to impulsive load, not to dead load. They are good forms to use in such cases as the so-called "marine type" of connecting rod, where the bolts are subjected to considerable shock.

For cylinder head bolts, and other cases where a tight joint is the main consideration, this form of bolt may be entirely unsuited.

Professor Sweet prepared, for tests, some bolts such as are used in the connecting rod of the Straight Line Engine ; of these, half were solid (ordinary form) bolts, and the other half were of the form shown in Fig. 37.

Tests of a pair of these bolts, one of each kind, showed an elongation at rupture of .25″ for the solid bolt, which broke in the thread ; while the drilled bolt elongated 2.25″, or 9 times as much, and it broke through the shank, the net section of which was a trifle less than that at the bottom of the threads. Drop tests showed similar results. These tests indicate the superior ultimate resilience of the reduced shank bolts.

60 Friction and Efficiency of Screws and Nuts.—When

a bolt is screwed up under load a torsional stress is produced in it, due to the frictional resistance overcome at the threads. If, in screwing up the bolt, pressure is produced between the members connected, their reaction may cause a considerable initial tension in the bolt; in fact, this initial tension due to screwing up is frequently much greater than that due to the external ("useful") load. The above-mentioned stresses are much affected by the friction of the threads and of the nut on its seat; for this reason the friction of screws is considered at this point. Read Unwin, § 83, as far as the bottom of page 149.

Referring to Fig. 87 (Unwin), it is to be noted that the force Q is due to the pull on the wrench. Of this pull, one portion is expended in overcoming the friction of the nut on its seat, while the remainder, reduced to the equivalent force acting at the mean radius of the thread, is this force Q. The actual pull on the wrench (neglecting friction of the nut on the seat) is Q multiplied by the mean radius of the bolt and divided by the effective radius of the wrench.

From the consideration that the "input" of energy must equal the "output" (including frictional losses), the following relations are deduced from examination of Fig. 87 (Unwin):

$$Q \cdot bc = P \cdot ac + F \cdot ab, \qquad (1)$$

$$\text{or } Q \cos \alpha = P \sin \alpha + F, \qquad (1')$$

because $bc : ac : ab :: \cos \alpha : \sin \alpha : 1$,

in which α = the inclination of the screw threads = angle abc. Also, the friction equals the normal pressure between the sliding surfaces multiplied by the coefficient of friction, or $F = R\mu$. From the relation that the sum of the vertical forces equals zero,

$$P = R \cos \alpha - F \sin \alpha = R(\cos \alpha - \mu \sin \alpha) \qquad (2)$$

$$\therefore \ R = \frac{P}{\cos \alpha - \mu \sin \alpha} \qquad (3)$$

Equation (1′) may now be reduced to the following:

$$Q \cos \alpha = P \sin \alpha + R\mu = P \sin \alpha + \frac{P\mu}{\cos \alpha - \mu \sin \alpha}$$

$$= P\left[\frac{\sin \alpha \cos \alpha + \mu(1 - \sin^2 \alpha)}{\cos \alpha - \mu \sin \alpha}\right] = P \cos \alpha \left[\frac{\sin \alpha + \mu \cos \alpha}{\cos \alpha - \mu \sin \alpha}\right] \qquad (4)$$

$$\therefore \quad Q = P\left(\frac{\sin \alpha + \mu \cos \alpha}{\cos \alpha - \mu \sin \alpha}\right) \qquad (5)$$

Since the circumference of the screw (πd) is to the pitch (p) as bc is to ac, or as $\cos \alpha : \sin \alpha$, equation (5) may be written thus,

$$Q = P\left(\frac{p + \mu \pi d}{\pi d - \mu p}\right) \qquad (5')$$

This last expression is eq. (5) of Unwin, § 83.

The coefficient of friction, μ, is equal to the tangent of the angle of repose (ϕ) of the two surfaces in contact, or $\mu = \tan \phi$. Dividing numerator and denominator in eq. (5) by $\cos \alpha$, and putting in $\tan \phi$ for μ,

$$Q = P\left(\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi}\right) = P \tan (\alpha + \phi). \qquad (6)$$

Inspection of eq. (1′) shows that for a frictionless screw $(F = 0)$,

$$Q \cos \alpha = P \sin \alpha \text{ or } Q = P \frac{\sin \alpha}{\cos \alpha} = P \tan \alpha ;$$

while eq (6) shows that with friction $Q = P \tan (\alpha + \phi)$.

Hence the effect of friction is to require an expenditure of energy equivalent to screwing up a frictionless bolt with threads inclined at an angle $(\alpha + \phi)$; while the useful work actually performed with this expenditure of energy is only that due to a screw of inclination α.

With *triangular* thread screws, the normal pressure at the threads is greater than with square threads; hence the friction at the threads is greater, other things being equal. In Fig. 38 the normal pressure for a square thread is indicated by R, while the resultant normal pressure for triangular thread is $R' = R \sec \theta$, in which θ = half the angle between the adjacent faces of a thread. R'' represents the radial crushing action on the thread of the screw, and its equal and opposite reaction tends to burst the nut. With 60° angular thread, as in the Sellers' system, or the common "V" thread, $R' = R \sec 30° = 1.15\, R$. The friction is increased directly as the normal pressure; or it is about 15 per cent. greater in the 60° angular thread than in the square thread.

As the friction, F, is $R\mu \sec\theta$ for triangular threads, equation (2) may be written thus:

$$P = R\cos\alpha - R\mu\sec\theta\sin\alpha \quad \therefore \quad R = \frac{P}{\cos\alpha - \mu\sec\theta\sin\alpha} \qquad (7)$$

Equation (1′) then gives,

$$Q\cos\alpha = P\sin\alpha + R\mu\sec\theta = P\sin\alpha + \frac{P\mu\sec\theta}{\cos\alpha - \mu\sec\theta\sin\alpha}$$

$$= P\left[\frac{\sin\alpha\cos\alpha + \mu\sec\theta\,(1 - \sin^2\alpha)}{\cos\alpha - \mu\sec\theta\sin\alpha}\right]$$

$$= P\cos\alpha\left(\frac{\sin\alpha + \mu\sec\theta\cos\alpha}{\cos\alpha - \mu\sec\theta\sin\alpha}\right)$$

$$\therefore \quad Q = P\left(\frac{\sin\alpha + \mu\sec\theta\cos\alpha}{\cos\alpha - \mu\sec\theta\sin\alpha}\right) = P\left(\frac{p + \mu\pi d\sec\theta}{\pi d - \mu p\sec\theta}\right) \qquad (8)$$

Or, when $\theta = 30°$,

$$Q = P\left(\frac{p + 1.15\,\mu\pi d}{\pi d - 1.15\,\mu p}\right) \qquad (9)$$

This last is the same as eq. (6) of Unwin, page 150. The portion of § 83 (Unwin) below eq. (6) may be omitted.

When the friction of the nut on its seat (or, of the thrust collar against its bearing) is considered, the force Q is the entire turning force reduced to its equivalent acting at the mean radius of the threads. Hence for square threads,

$$Q\pi d = P\pi d\left(\frac{p + \mu\pi d}{\pi d - \mu p}\right) + P\mu_1\pi d_1,$$

in which d_1 is the mean friction diameter of the nut (or collar) and μ_1 is the coefficient of friction of this nut on its seat. If the ratio of d_1 to d be called a,

$$Q = P\left(\frac{p + \mu\pi d}{\pi d - \mu p} + a\mu_1\right) = P\left(\frac{\sin\alpha + \mu\cos\alpha}{\cos\alpha - \mu\sin\alpha} + a\mu_1\right) \qquad (10)$$

For triangular threads, terms containing μ in eq. (9) should be multiplied by the secant of half the angle between adjacent faces of the thread, or by $\sec\theta$, as in eqs. (8) and (9) above; but the term containing μ_1 *is not to be so multiplied* because the friction of the nut on the seat is not affected by the form of the thread.

For the ordinary standard forms of nuts the mean friction diam-

eter of the nut may be assumed at about $1\frac{1}{3}$ times the diameter of the bolt; or $d_1 = \frac{4}{3} d$, hence, in eq. (9), the coefficient $a = \frac{4}{3}$. With the Sellers' system of threads, or the most usual form of full "V" threads, the normal pressure at the threads is 1.15 times the pressure for square threads, as already noted.

Dividing both numerator and denominator of the fractional part of eq. (10) by $\cos \alpha$, and inserting the values of a and of $\sec \theta$ as just assigned, for standard screw threads,

$$Q = P\left(\frac{\tan \alpha + 1.15\,\mu}{1 - 1.15\,\mu \tan \alpha} + \tfrac{4}{3}\,\mu_1\right) \qquad (11)$$

In the Sellers' system, α varies from about $2°\ 45'$ in a $\frac{1}{2}$ inch screw to $1°\ 45'$ in a 3 inch screw; or $\tan \alpha$ varies from .049 to .0303 in this same range. If μ be taken at .15 and μ_1 at .10 it appears that Q varies from .356 P with a $\frac{1}{2}''$ screw to .337 P with a $3''$ screw. The coefficients of friction will vary much more than this, so it may be assumed that for the ordinary range of bolts,

$$Q = .345\,P \text{ (approximately.)} \qquad (12)$$

If friction could be entirely eliminated eq. (1) would become

$$Q \cdot ab = P \cdot ac + 0\text{; or } Q \pi d = P p$$

$$\therefore\ Q = P \frac{p}{\pi d} = P \tan \alpha.$$

Then for a standard 1 inch bolt *with no friction*, $Q = .04\,P$; while with the above assumptions as to friction, $Q = .345\,P$, approximately. The ratio of these two values of Q gives an expression for the efficiency of the screw and nut, and for the above conditions, this is about 11.5 per cent. It will be seen in the next article that this result agrees quite closely with certain direct experiments.

The relation between the load on a screw, P, and the tangential turning force, Q, is given by eq. (5) of this article, when the screw is being turned so as to produce a motion of the loaded member opposite to the direction of the force P. This direction of motion will be designated as "hoisting," though the load may not be actually moved upward by this action. With the usual proportions, the load will not drive the screw backward when the turning

force Q ceases to act, *i. e.*, the screw mechanism will not "overhaul;" but some force opposite in direction to Q must be applied to "lower." Screws may be so proportioned that they will overhaul, but this condition is not usual.

Referring again to Fig. 87, Unwin, it is apparent that if the load is being lowered the friction, F, would be opposite in direction to that indicated, as the friction always opposes the relative motion of the members between which it acts. It will be assumed that the direction of Q is as yet unknown but its direction in hoisting (that indicated in Fig. 87) will be called positive. As stated above, with the usual proportions of screws Q is negative. The equation for lowering, which corresponds to eq. (1) of this article, becomes

$$Q \cdot bc = P \cdot ac - F \cdot ab \tag{13}$$

or

$$Q \cos \alpha = P \sin \alpha - F \tag{13'}$$

in which it remains to be seen whether Q is positive or negative. It is evident, however, that the work $P\ ac$ which measures the tendency of the load to run down is opposite in sign to $F \cdot ab$ which resists overhauling. As before, $F = R\mu$, and from the equality of the vertical components of the concurrent forces,

$$P = R \cos \alpha + F \sin \alpha = R(\cos \alpha + \mu \sin \alpha) \tag{14}$$

From eqs. (13′) and (14)

$$Q \cos \alpha = P \sin \alpha - R\mu = P\left(\sin \alpha - \frac{\mu}{\cos \alpha + \mu \sin \alpha}\right)$$

$$\therefore Q = P\left(\frac{\sin \alpha - \mu \cos \alpha}{\cos \alpha + \mu \sin \alpha}\right) \tag{15}$$

To investigate the sign of Q, assume that it equals zero, as it will for a certain relation of α and μ. Then

$$P\left(\frac{\sin \alpha - \mu \cos \alpha}{\cos \alpha + \mu \sin \alpha}\right) = 0 \quad \therefore \sin \alpha = \mu \cos \alpha$$

$$\frac{\sin \alpha}{\cos \alpha} = \tan \alpha = \mu = \tan \phi \tag{16}$$

since the coefficient of friction (μ) equals the tangent of the angle of repose (ϕ). Hence, when $Q = 0$, $\alpha = \phi$, or the inclina-

tion of the thread helix equals the angle of repose. The angle of repose depends upon the nature of the materials used, their condition as to finish, and the lubrication.

If $\alpha > \phi$, $\tan \alpha > \mu$ $\therefore$ $\sin \alpha > \mu \cos \alpha$ (multiplying both sides of the inequality by $\cos \alpha$); hence the numerator of eq. (15) $(\sin \alpha - \mu \cos \alpha)$ is positive, and Q is positive. It is evident that this must be so, for if $Q = 0$ when $\alpha = \phi$, Q must be positive for an inclination of the screw greater than the angle of repose to prevent overhauling.

If $\alpha < \phi$, $\tan \alpha < \mu$ $\therefore$ $\sin \alpha < \mu \cos \alpha$; hence $\sin \alpha - \mu \cos \alpha$ is negative and Q must be negative; or the force P will not over haul the mechanism when the inclination of the helix is less than the angle of repose, and some force $(-Q)$ must assist the tendency of P to overhaul before lowering can actually occur.

Since Q is usually negative, it is convenient to write $Q' = -Q$, when eq. (15) reduces to

$$Q' = P\left(\frac{\mu \cos \alpha - \sin \alpha}{\cos \alpha + \mu \sin \alpha}\right) \tag{17}$$

Or, substituting p for $\sin \alpha$, and πd for $\cos \alpha$,

$$Q' = P\left(\frac{\mu \pi d - p}{\pi d + \mu p}\right) \tag{17'}$$

The friction of the nut or thrust collar on its seat increases work to be overcome in lowering as well as in hoisting, and it is to be added to eq. (17′) above. Including this resistance

$$Q' = P\left(\frac{\mu \pi d - p}{\pi d + \mu p} + a \mu_1\right) \tag{18}$$

in which a is the ratio of mean friction diameter of the nut (or collar) to the mean diameter of the screw threads, and μ_1 is the coefficient of friction of the nut on its seat, as before.

For a "V" thread μ should be multiplied by the secant of half the angle between the adjacent surfaces of the threads; but as the thread most commonly used for transmission of energy is either a square thread, or one approximating it, equation (18) of this article will ordinarily apply in computations relating to lowering.

61. Initial Tension in Bolts due to Screwing Up. [Unwin,

§ 85, page 151,] It is assumed by Unwin that the radius of wrench will usually be about 15 times the diameter of the bolt, and that the heaviest ordinary pull of the workman will be about 30 lbs. On this basis, he estimates that the initial tension on a bolt due to screwing up will be about 2500 lbs., regardless of the size of the bolt; although it is stated in this connection that experience teaches the mechanic in what case a heavy pressure may be applied with safety. While this view seems plausible, it is probable that the initial tension due to screwing a nut up tight is usually very much greater than 2500 lbs.

A series of experiments was made in the Sibley College Laboratory, a few years ago, to directly determine the probable load produced in standard bolts when making a tight joint. The sizes of bolts used were $\frac{1}{2}''$, $\frac{3}{4}''$, $1''$ and $1\frac{1}{4}''$. One set of experiments was made with rough nuts and washers, and another set with the nuts and their seats on the washers faced off. A bolt was placed in a testing machine, so that the axial force upon it could be weighed after it was screwed up. Each of twelve experienced mechanics was asked to select his own wrench and then to screw up the nut as if making a steam-tight joint, and the resulting load on the bolt was weighed. Each man repeated the test three times for every size of bolt, and each had a helper on the $1''$ and $1\frac{1}{4}''$ sizes. The sizes of wrenches used were $10''$ or $12''$ on the $\frac{1}{2}''$ bolts up to $18''$ and $22''$ on the $1\frac{1}{4}''$ bolts. The results were rather discordant, as should be expected; the loads in the different tests were rather more uniform, as well as higher, with the faced nuts and washers. The general result indicates: (a) that the initial load due to screwing up for a tight joint varies about as the diameter of the bolt; that is, a mechanic will graduate the pull on the wrench in about that ratio. (b) That the load produced may be estimated at 16,000 lbs. per inch of diameter of bolt, or

$$P_1 = 16{,}000\, d \qquad (1)$$

in which P_1 is the initial load in pounds due to screwing up, and d is the nominal (outside) diameter of the screw thread. This value of P_1 is rather above the average for the tests; but it is con-

siderably below the maximum, and it is probably not in excess of the load which may reasonably be expected in making a tight joint.

If the initial load due to screwing up be divided by the cross-sectional area of the bolt at the bottom of the threads, the initial intensity of the tensile stress is obtained. The above experiments indicate that this intensity of stress varies, approximately, inversely as the nominal diameter (d) of the bolt; and that it may frequently equal or exceed

$$f = \frac{30{,}000}{d} \text{ lbs. per sq. in.} \qquad (2)$$

In addition to this tensile stress there is a considerable twisting action on the bolt. Equation (2) would give a stress of 60,000 lbs. per square inch on a ½ inch bolt; and this result is substantiated by the fact that steel bolts of this size were broken in the course of the experiments. It also agrees with common experience which forbids the use of screws as small as ½ inch for cases requiring the nuts to be screwed up hard.

In these experiments, the average effective lever arm of the wrench was not far from 15 times the diameter, or 30 times the radius, of the screw; hence, if it be assumed that the turning force acting at the radius of the screw is $Q = .345\,P$, as in eq. (12) of art. 60, the force applied at the wrench is, in pounds, about

$$Q_1 = \frac{Q}{30} = \frac{.345\,P}{30} = \frac{.345 \times 16{,}000\,d}{30} = 180\,d,$$

instead of 30 lbs for all sizes of bolts, as assumed by Unwin.

Unwin's assumption would indicate that the intensity of stress varies inversely as the square of the diameter, instead of inversely as the first power of the diameter. On the other hand, a common practice in designing is to take a low working stress (large factor of safety) "to allow for the stress due to screwing up." This procedure implies that the *intensity of initial stress* is the same for all sizes of screws, which does not seem to be justified either by reason or experience.

The above discussion indicates that the factor of safety should

be increased as the size of screw decreases, and of course this factor should be varied with the conditions of the case, as in some applications the nuts are much more apt to be screwed up hard than in others.

A set of experiments were made by Mr. James McBride (Trans. A. S. M. E., Vol. XII, page 781) which show that the factor of safety, as bolts are frequently used, is very low, even with a very moderate external load. One case cited by Mr. McBride indicates that the stress due to screwing up a 3⅛ inch bolt was nearly one-half the ultimate strength, or probably very near the elastic limit, His direct determinations of the efficiency of a standard 2 inch screw bolt shows an average of only 10.19 per cent. It is probably this low efficiency which saves many screws from being broken, as the frictional loss reduces the tension produced in the bolt by screwing up. The excessive friction makes the screw bolt a useful fastening, as it reduces the tendency to "overhaul" or unscrew.

62 Resultant Stress on Bolts due to Combined Initial Tension and External Load.—It was shown, in article 61, that bolts may be subjected to a high tensile stress by screwing up the nuts. The question often arises as to the effect of the combined action of this initial tension and the external, or useful, load. It is stated by some that the resultant load on the bolt is simply the sum of the initial and the external loads Others contend that the application of the external load does not change the stress in the bolt, unless this external load exceeds the initial load due to screwing up ; that is, that the resultant load is equal to either the initial load alone, or to the external load alone, whichever is the greater.

Neither of these views is entirely correct for conditions attained in practice. They represent the extreme limiting cases and every actual case lies between them.

If the bolt itself could be absolutely rigid while the members forced together in screwing it up yielded under pressure, the total load on the bolt would be equal to the sum of the initial load and the external load. If, however, the members pressed together

were absolutely rigid, only the bolt yielding, the total (resultant) load on the bolt would be the initial load alone, or the external load alone, whichever is the greater.

The first of the above conditions is approached by the arrangement shown in Fig. 39. Screwing up the nut compresses the spring interposed between A and B. Assume that an axial force of 2000 pounds will compress this spring 1 inch; then if the nut is screwed up till the spring is 2 inches shorter than its free length, the load on the bolt, due to screwing up, must equal the reaction of the spring, or 4000 lbs. Assume, also, that the extension of the bolt under this screwing up action, or under the initial load of 4000 lbs., is .02 inch. Now, if an external axial load of say 2000 lbs. be applied to the eye at the bottom of B, this added load would tend to further increase the length of the bolt by about .01 inch; but this further extension of the bolt would reduce the compression on the spring by a corresponding amount and thus slightly diminish the spring reaction. With such great difference between the rigidity of the bolt and of the connected members, the load on the bolt becomes practically the sum of the initial and the external loads, but the resultant load is necessarily somewhat less than this sum in any possible case.

The arrangement shown in Fig. 40 is one which approaches the other limiting case mentioned above. Suppose the bolt to be a spring which is subjected to an axial load of 4,000 lbs. in screwing the nut up 2 inches, and that the corresponding yielding of the member B is .02 inch. The initial load on the bolt (which is the spring in this case) is 4,000 lbs., and the pressure between the contact surfaces of A and B is equal to it. If an external axial load be now applied to the eye in B, the pressure between the contact surfaces is reduced by an amount nearly equal to this external load. But unless the external load exceeds the initial load, the bolt will not elongate enough to separate these contact surfaces and entirely remove the pressure between them, because the load on the bolt (spring) cannot change without changing the length of the bolt, and with the above data the bolt would have to stretch an additional .02 inch (equal to the

initial yielding of the connected members) before the contact surfaces would be entirely relieved of pressure. It therefore appears that the addition of an external load in this case does not materially affect the resultant tension on the bolt as long as this external load does not exceed the initial load. If the external load is greater than the initial load (say 6,000 lbs.) the elongation of the bolt increases (to 3 inches); the resultant load on the bolt will be simply the external load alone, because the latter is sufficient to entirely relieve the pressure produced between the contact surfaces in screwing up.

In all ordinary practical cases the difference in rigidity between the bolt and the connected members is much less than in the extreme conditions considered above. The resultant load on a bolt may be anything between the *sum of the initial and the external loads* as a maximum, and the *greater of these two loads alone* as a minimum. This resultant load approaches the maximum limit when the bolts are rigid relative to the connected members as in Fig. 41; and this resultant approaches the minimum limit when the bolts are relatively yielding, as in Fig. 42. In any particular case the designer can tell which limit is the more nearly approached, and he should be governed accordingly.

The "Locomotive" (Nov., 1897) contains an excellent article on the resultant load on bolts, and a relation is derived from which the following method of treatment has been developed: The application of this method depends simply upon the *ratio* of the yield of the connected members to the yield of the bolts. It will usually not be difficult to assign a sufficiently close value to this ratio, even when the actual magnitudes of yielding are unknown; in fact, only a rough approximation to the value of this ratio is necessary. Let this ratio be called y and let $\frac{y}{y+1} = x$; call the initial load on the bolt due to screwing up P_1; the external (useful) load P_2; and the total (resultant) load P. Then it can be shown that

$$P = P_1 + x P_2. \qquad (1)$$

If the yield ratio (y) is known, the value of x is at once found by

the above relation of x and y. If the yield of the connected members is between 1 and 5 times that of the bolt, the resultant load is equal to the initial load added to from 0.5 to 0.8, the external load. If a tight joint it made with short rigid bolts or studs, connecting flanges which are separated by an elastic packing, or with a metal contact at some distance from the centre line of the bolts, as indicated in Fig. 41, the applied load is an important consideration since the value of y is relatively great.. In some other cases the external load may be a minor consideration as affecting the strength of the bolt.

When the conditions are such that the nut is not apt to be screwed up hard, that is when the initial load may be safely neglected, design for the external load alone.

The following suggestions may serve as a guide in practical problems involving the resultant load on bolts when the initial load due to screwing up is apt to be considerable:

(*a*) If the bolt is manifestly very much more yielding than the connected members, design the bolt simply for the initial load or for the external load, which ever is the greater.

(*b*) If the probable yield of the bolt is from one-half to once that of the connected members, consider the resultant load as the initial load plus from one-fourth to one-half the external load.

(*c*) If the yield of the connected members is probably four or five times that of the bolts, take the resultant load as the initial load plus about three-fourths the external load.

(*d*) In case of extreme relative yielding of the connected members, the resultant load may be assumed at nearly the sum of the initial and external loads.

63. Proportions and Forms of Bolts and Nuts. [Unwin, §§ 86–88.]

64. Locking Arrangements for Nuts. [Unwin, § 89.]

65. Bolting Plates. [Unwin, §§ 90–91.]

66. Joint Pins. [Unwin § 92.]

The sections of the pin should be checked for resistance to shearing, like a double shear rivet. The eyes, or bosses, in which the pin bears, should be checked against tearing out. In general,

the strength of each of these parts should at least equal that of the rod which transmits the load to the joint. In cases where the motion at the joint is considerable, the proportions necessary to secure sufficient area of the bearing to avoid undue wear may give an excess of strength. The proportions of bearings and journals will be treated in the next chapter.

67. Keys. [Unwin §§ 93 to 98.]

Sunk keys may be divided into three classes:—Flat keys; square keys; and feather keys. The ordinary form of key for heavy work is the flat key, as shown by Fig. 112 C (Unwin). This is the form commonly used in engine work and construction of mill machinery, etc. As it is tapered top and bottom and driven in hard it is tightest on these faces, but it should also be fitted on the parallel sides. While this key resists shearing, it really acts as a diagonal strut, in transmitting force, as indicated in Fig. 43 With the usual proportions, the severest action on the key is the crushing forces at the bearing surfaces a, a'. The resistance to relative rotation of the connected members is due to the resistance of the key to crushing and shearing, and to the friction between the hub and shaft at b (Fig. 43). The effect of the taper of the key is to somewhat distort the hub and to throw it out of exact concentricity with the shaft; this latter effect being greater because the hub is usually bored sufficiently large to be easily moved to place along the shaft. However, with the usual dimensions of hub, the springing of the work is not apt to exceed limits which are admissible in most heavy machines.

The tendency of the hub to work loose on the shaft, especially when the forces acting are continually reversed in direction, can be largely overcome by placing two keys "quartering", as shown in Fig. 113 (Unwin).

Unwin states that a taper of 1½ inch per foot corresponds to about the angle of repose for oiled steel and iron surfaces. Such a steep taper of the key would not give a good grip of the hub on the shaft, and the shock incident to operation would be very liable to loosen the key. The usual taper of keys is about ⅛th inch per foot.

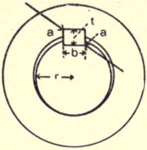


Fig. 43.

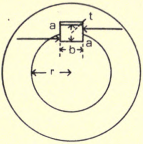


Fig 44

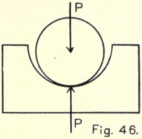


Fig. 46.

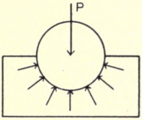


Fig. 47.

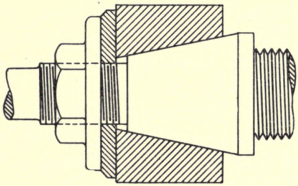


Fig. 45.

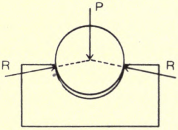


Fig. 48.

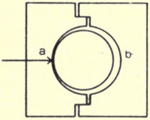


Fig. 49 (a).

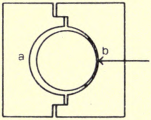


Fig. 49 (b).

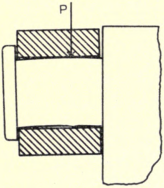


Fig. 50.

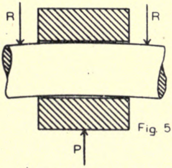


Fig. 51.

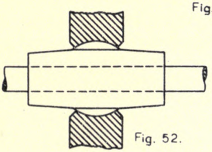


Fig. 52.

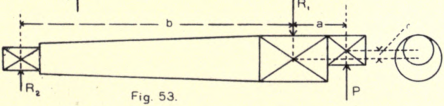


Fig. 53.

If the thickness (t), or depth, of the key is too great, the shaft is unduly weakened by the deep key seat. In practice, the following proportions are usual ones: $b = \frac{1}{4}\, d$; $t = \frac{1}{2}\; b = \frac{1}{8}\, d$; in which $d =$ the diameter of the shaft, $b =$ the breadth, and $t =$ the thickness or depth of the key. In application, the dimensions would be to the nearest $\frac{1}{16}$th inch. The above proportions are suitable for the key of a main driving pulley or gear; that is for transmitting a twisting moment up to full capacity of the shaft.

In cases where light pulleys or gears are carried on large shafts, the dimensions of the keys may often be much less than those assigned by the above formulas. The form of key above discussed resists axial motion of the hub along the shaft by friction between the contact surfaces.

Where the power is distributed through pulleys or gears, each carrying only a small portion of the load, set screws are frequently used as a substitute for keys. These are convenient, but of doubtful holding capacity under any but quite light loads. If they slip, the shaft is apt to be burred up so that it is difficult to remove the pulley.

If the twisting moment transmitted through the key is T, the pressure against the bearing faces is $P = T \div r$; r being the radius of the shaft. The area of each bearing face is $\frac{1}{2}\, t\, l$, in which t is thickness of the key and l is the length of hub. The crushing stress per unit of area of bearing face is $f_c = 2\, P \div t\, l$.

In members of machine tools, and in other similar cases, the eccentricity of the hub which is liable to result from driving a taper flat key may be sufficient to appreciably impair the accuracy of operation. Under such conditions, particularly if the service is not severe, *square keys* may be advantageously substituted for flat keys. Fig. 44 illustrates the true square key. It should be fitted tightly on the parallel sides $a\, a$, but should not be tight at top and bottom. It is evident that a key fitted in this way has no tendency to throw the hub out. The hub should be bored so as to fit the shaft closely, though not tight enough to require heavy driving in putting the hub in place.

When a square key is properly fitted the stress upon it from the load transmitted may be taken as a simple shear $= f_s = P \div b\, l$;

in which *b* is the breadth and *l* the length of the key. The bearing faces are subjected to a crushing action, similar to that with a flat key. In this type of keys, *b* is usually equal to *t*.

Feather keys are used when the hub occupies different positions on the shaft; as in the driving gear of drill press spindles. In such cases a key fixed in the hub slides in a long keyway, or spline, cut in the shaft; or a long key, or "feather," is fixed in the shaft, and the keyway of the hub slides along this "feather." Of course the fit of the key in the groove along which it slides must be a "sliding fit." If the axial motion under load is great, the bearing surface of the sliding elements should be sufficient to avoid excessive wear.

VII.

JOURNALS AND BEARINGS; THRUST BEARINGS; GUIDES.

68. Outline of the Functions and Operation of Bearings. [Unwin, §§ 104–105, page 181].

The leading considerations in the design of journals and bearings, or other guiding elements which by sliding upon each other constrain the motion between two members, are: Form; dimensions; materials; lubrication.

Where the two members are required to have relative rotation in parallel planes, the journal and bearing must have contact surfaces which are a pair of surfaces of revolution, with a common axis coinciding with the axis of relative rotation. The most usual form for such constraining elements is the right cylinder; frequently provided with collars to limit the end play. In some cases no appreciable end motion is permissible, and in few cases would this play be more than a small fraction of an inch. The bearing is commonly split along its length with provision for closing it on the journal to compensate for wear. In some cases the contact surfaces are conical with a means of moving the journal axially (relatively to the bearing) to take up the wear. A method of taking up wear in conical bearings is indicated by Fig. 45.

Two members not moving in parallel planes are sometimes connected through bearings having spherical surfaces, or by what is commonly called a "ball and socket joint."

If the desired relative motion is a translation, appropriate guiding surfaces (having elements parallel to the line of motion) are used.

The strength of a journal depends upon its form, dimensions, material, and method of support. For example, an overhanging crank-pin, or end journal, may be treated as a cantilever

of circular section, in which the working strength is proportional to the cube of the diameter, inversely as the length, and directly as the safe stress for the material. A journal of similar form and dimensions, if supported at both ends, is evidently much stronger than this cantilever.

When a journal runs in its bearing, the frictional resistance results in heat and in wear of both members. In some cases, as in long lines of transmission shafting, the most important effect of friction is the resulting loss of energy. In other cases, as in engine crank-pins and eccentrics, the danger of heating and of resultant injury to the bearing surfaces is paramount. In still different classes of bearings, as those of grinding lathes, and other light machine tools for producing very accurate work, the most objectionable result of friction is the change of form through wear, which affects the accuracy of constrainment and thus impairs the quality of the product of the machine. The danger from overheating and the loss of energy through friction are usually secondary considerations in this last named class of bearings.

While analysis may suggest general relations between lengths and diameters of journals applicable to these special classes, the design of such bearings for permanence or durability must be based very largely upon experience, and the sizes and proportions in general use are the result of a process of evolution.

Frictional loss is mainly dependent upon the velocity of rubbing at the journal surface, the bearing pressure, and the lubrication. With perfect lubrication (which is very difficult to maintain) the friction is influenced but little, if at all, by the materials of the journal and bearing, except as the smoothness of the surfaces is affected by the nature of the materials. But in case of failure of lubrication, these materials have an important influence on the consequences. A film of oil between the journal and bearing completely separates their surfaces when ideal lubrication exists. No surface is absolutely smooth, and the small points and irregularities projecting from either surface tend to pierce the oil film and thus to abrade the opposite surface. The smoother the surface the less will be this tendency, and the thinner the oil film

can become without permitting metallic contact. Aside from the susceptibility to smooth finish, the nature of the materials composing the journal and bearing have little effect on the action as long as a good film of oil is maintained. If, however, the lubrication fails, partially or completely, and the condition of metallic contact is established, one or both of the members may be seriously "cut" before the machine is stopped and proper conditions are restored.

As stated above, the friction is quite independent of the materials of the journals and bearings (aside from the influence of finish or polish) as long as the lubrication is properly maintained. Different bearing materials seem to be capable of carrying about the same intensity of pressure with good lubrication, provided the maximum intensity of pressure is below that at which the material will crush, or flow. When the bearing is subject to considerable shock or "hammering," a soft metal bearing face may have the oil grooves closed up, or become otherwise so deformed that the distribution of the oil will be impeded and lubrication interrupted. It happens, not infrequently, that bearings run hot from such cause. Of course the material used should not yield appreciably under any compressive action to which it is apt to be subjected in service.

It appears that lead and zinc alloys are more liable to corrosion from animal or vegetable acids than are tin and copper ; and this probably accounts in part for the favor in which genuine Babbitt metal (tin 88 per cent,, antimony 8 per cent., copper 4 per cent.) has been held. Corrosion roughens the surface, and may cause serious heating. The purely mineral oils are less apt to act upon the material in this way, and numerous bearing materials containing lead have found extensive use in recent practice.

In the event of failure of lubrication, the metallic surfaces run in actual contact. If the bearing material has a firmer and stronger structure than the journal, the latter is most abraded, and the particles removed from it tend to heap up at one point on the stationary bearing, cutting the journal more and more deeply at each turn. If, on the other hand, the bearing material yields

more readily than that of the journal the material removed adheres to the revolving member and rotates with it, thus scoring the surfaces less than with the opposite conditions. Hence, it is generally safer to use a bearing metal which is softer than the journal. The lining of a bearing is generally more cheaply replaced than the journal, and it is therefore desirable to have such injury as occurs confined to the former, as far as it can be. These reasons indicate why it is not usually good practice to make journal and bearing of similar materials. There are exceptions to the rule that the bearing should be the softer material.

In cases of low velocity of rubbing, cast iron has often been found to give good results when running on cast iron. Cast iron piston rings in a cast iron steam cylinder have usually been found to furnish the best combination. In spindles of milling machines and grinding machines the bearing is frequently a hardened steel bushing, while the journal is of softer steel. These bearings are often conical (see Fig. 45), and the wear, if concentrated on the rotating shaft, takes place quite uniformly all around ; but if the wear were mainly in the stationary bearing it would occur on the side of greatest pressure. It will readily appear that the compensation for wear affects the alignment of the spindles less if this wear is uniformly distributed around the journal.

Bronze, either with or without a "white metal" lining, is much used for bearings. The chief advantages of bronze for a bearing surface, proper, are its resistance to compression and shock, and less liability of injuring a wrought iron or steel shaft than an iron bearing surface When bronze is used in a bearing with a "Babbitt" or other lining, the bronze is not the true bearing material, unless the lining metal melts and runs out. The justification for use of bronze rather than cast iron in such cases (as in connecting rod "brasses") is in its superior strength under shock to cast iron, and the ease with which it is fitted up ; together with the reduced danger to the shaft in case the bearing ever becomes so hot as to melt out the "Babbitt" before the machine is stopped.

69. **General Considerations on Friction.**—[Unwin, §§ 106 to 109]. If the journal has a somewhat smaller diameter than the bearing, as in Fig. 46, the contact is theoretically (except for the influence of the oil film) along a line; and the load on the bearing is P, equal to the load carried by the journal. Of course, small wear of the bearing would result in distribution of the load over a finite area on the bearing, but the load on the bearing would remain substantially equal to P with any moderate amount of wear. If the journal and bearing originally have the same diameter, as indicated by Fig. 47, the bearing pressure would theoretically be a uniformly distributed normal pressure equal to $\frac{\pi}{2} P$. A bearing could not run long under this condition, for the oil film could not be maintained with such a fit. The condition indicated by Fig. 48 is that in which the journal is initially of larger diameter than the bearing. In this last case, the bearing pressure, $R R$, might be infinity except for the yielding of the members and wear; but such a bearing would rapidly change to the form shown in Fig. 47, and would tend to approach that of Fig. 46. However, the surfaces might be seriously injured during this change, and the successful operation of a bearing ordinarily requires that it be fitted up with a slightly larger diameter than the journal. It is, therefore, justifiable to treat the load on the bearing as equal to P, and the friction at the bearing as μP; μ being a special coefficient for bearings, as explained on page 183 of Unwin; though the oil film distributes the pressure.

70. **Outline of the Theory of Lubrication.**—[Unwin, § 110]. Oil applied to the surface of a rotating journal tends to adhere to this surface and to be carried around with the journal. The adhesion of the oil to the journal is the means of transferring it from the side of least pressure, where it should be introduced, to the loaded side of the bearing. Of course the oil, upon coming into contact with the stationary surface of the bearing, adheres to this surface also; so that a layer of oil next the journal tends to revolve, and a layer next the bearing tends to remain stationary. Owing to the cohesive action between the particles of the oil

(viscosity) a resistance is offered to this relative motion of its seprate particles, and the friction of a well lubricated bearing is due, mainly, to this fluid resistance. The globules of oil tend to roll between the two surfaces like balls in a ball bearing, though the action is more like that which might be imagined to exist if the balls were strongly magnetized. The rotation of the layer of oil next to the journal is retarded somewhat on account of the adhesion of the oil to the stationary surface of the bearing and by the cohesion of the intermediate particles. On the other hand, the adhesion of the oil to the journal and the cohesion between the particles tend to carry the oil film around with the journal. Notwithstanding the pressure on the loaded side of the bearing, some of the oil will be dragged along with the journal into the curved wedge shaped space between the journal and bearing (see Fig. 46), if the intensity of bearing pressure is not too great for the viscosity of the oil used. A very close fitting bearing, as in Fig. 47, would not admit the oil readily, and in general the edges of the bearing should be rounded or chamfered. If the intensity of bearing pressure is low enough for the oil used (remembering that the viscosity of the oil becomes reduced as the bearing becomes warm) the metal surfaces are kept separated by the film of oil. As shown in the reports of Tower's experiments, this pressure may be equivalent to 300 or 400 pounds per square inch of projected area on a bearing subjected to a constant load. If the pressure on the bearing is intermittent, the intensity of pressure may be much higher than these figures. In the case of the crank pin of the ordinary steam engine, in which the direction of pressure completely reverses during a revolution, pressures as high as 1,000 pounds per square inch of bearing are frequently carried. The conditions of the crank pin bearing during the opposite strokes of the piston is shown by Figs. 49*a* and 49*b*. In the former, the pressure tends to force the oil from the side *a* to the side *b*.; but the reversal of pressure on the return stroke tends to return the oil to side *a*. The sluggishness of the flow from one side to the other prevents the complete expulsion of the oil film from the pressure side during the short time of a single stroke;

while a similar intensity of pressure continuous in direction might expel the oil. The conditions are even more favorable at the crosshead pin, in horizontal engines, as the pressure and the direction of motion between the journal and bearing are both reversed at each stroke ; actions which tend to distribute the oil where needed. For this reason, and also because the velocity of rubbing (hence the tendency to heat) is less at the crosshead pin, the intensity of bearing pressure at this pin is usually considerable greater than that at the crank pin. These two pins carry substantially the same total load, but the intensity of pressure at either is this load divided by the projected area of its bearing, and the crosshead pin is usually considerably smaller than the crank pin. In vertical engines, the difficulty in introducing oil at the top of the crosshead pin results in less favorable conditions; because the motion of the journal is not sufficiently great to distribute the oil over the top bearing, where the pressure is applied on the down stroke.

If the intensity of bearing pressure is small, a light bodied oil can be used. From what has been said in regard to the friction being mainly due to the fluid resistance of the oil (with thorough lubrication), it will appear that a thin, fluid oil with a low intensity of pressure is generally desirable, on the score of reducing friction, especially for high speed bearings. However, in many cases of heavily loaded bearings unduly large surfaces might be required to secured a low intensity of pressure upon them. Furthermore, if the reduction of pressure is obtained by increasing the diameter of the journal, the velocity of rubbing is correspondingly increased, for a given rotative speed. The *friction* may thus be decreased by the larger diameter and lower pressure, which permits the use of lighter oil, while the *work of friction* (the product of the friction into the space passed over against this resistance) would not be decreased. It is this frictional work which measures the loss of energy. If the bearing pressure is reduced by increase of the length of the journal, the velocity of rubbing is not increased, but a limit to increase of bearing area by this means is imposed by considerations of strength and rigidity. The journal is often a beam of some form, the strength

and rigidity of which are decreased by increasing the length unless the diameter is correspondingly increased. Even if the journal is not in danger of breaking with such increase of its length alone, it may spring under its load enough to concentrate the pressure upon a small portion of the nominal bearing area and thus the maximum intensity of pressure on the bearing may be as great as, or even greater than, that due to a shorter journal. See Fig. 50, which indicates this action to an exagerated degree. A journal supported on both sides of the bearing (Fig. 51) can evidently have a greater length, relative to its diameter, than an overhung journal (Fig. 50), other things being equal. If the bearing is so mounted that it can swivel freely, and thus accommodate itself somewhat to the deflection of the journal (or shaft), as indicated in Fig. 52, the length can be greater relatively to the diameter than with a rigid bearing. Overhung crank pins of engines usually have lengths not exceeding 1¼ diameters: the main journals of engine shafts frequently have lengths equal to 2 diameters; and with the swivel bearings of ordinary lines of shafting the length of bearing is quite commonly 4 diameters. These proportions will be treated in a later section.

71. **Point of Introduction of Oil.**—[Unwin, § 110.] If the oil is applied nearly opposite the point of maximum pressure it finds ready admission; but the lubrication cannot be satisfactorily accomplished if the oil is introduced at, or near, the point of maximum load, unless it be forced in by a pressure exceeding in intensity the bearing pressure at the place of admission. Mr. John Dewrance, in a most valuable paper before the Institution of Civil Engineers (Great Britain) on "Machinery Bearings," gives the following rule: "The oil should be introduced into the bearing at the point that has to support the least load, and an escape should not be provided for it at the part that has to bear the greatest load." If this rule were always followed in construction, the elaborate system of oil channels seen in bearings would frequently be useless, or worse than useless. Direct channels from the oil hole to near the ends of the bearing, to distribute the oil latterly, would be sufficient for most cases, with the oil hole properly placed. The results quoted in

Unwin, page 188, also show the importance of adhering to this rule. Many instances have been observed of the oil bubbling up into the oil cup, when this simple rule has been neglected.

72. **Theory of Journal Proportions.**—[Unwin, §§ 112 to 114.] As pointed out by Professor Unwin in § 112, the proportions of journals as found in practice seem to agree better with the old assumption that μ=a constant (the law of dry friction) than with those laws which would be derived from Mr. Tower's experiments. The dimensions of practice are generally those found by experience to be desirable. For everyday running, the proportions must be adequate to the most unfavorable conditions which arise; not simply for the almost perfect lubrication which may be maintained in the laboratory. The common oil cup feed is, at its best, less effective than an oil bath, and it may fail entirely. At such a time the conditions of dry friction are reached if the derangement is not promptly detected and remedied.

In equation (3) [Unwin, § 112], the quantity μP is the friction (in pounds) and $\pi d N$ is the velocity of rubbing (in inches per minute). Hence, the work of friction per minute in ft. lbs. = $\mu P \frac{\pi d N}{12}$. This must be divided by J to express this energy in B.T.U. per minute. The surface through which this heat is dissipated is $\pi d l$, or it is proportional to the projected area = $d l$, and it is more convenient in calculations to use the projected area.

An increase of d increases the surface for dissipation of the heat; but it increases the velocity of rubbing, hence the heat to be dissipated, in the same proportion. It appears from eq. (4) [Unwin, § 112] that the heat developed per square inch of heat liberating area, and consequently the *temperature* attained, is independent of the diameter but inversely as the length of the bearing. It is also evident that the wear should be proportional to the friction times the velocity of rubbing, but this velocity, and the surface over which the wear is distributed, both vary as the diameter; hence the wear should be independent of the diameter. On the other hand, eq. (3) shows that the total heat developed, and therefore the actual *energy lost*, is directly proportional to the

diameter and independent of the length. It thus appears desirable to have as long a bearing as possible to secure cool running, with as small a diameter as possible to reduce lost work. These conclusions are subject to the following limitations, however: (a) If the diameter is too small, relatively to the length, the deflection of the journal may concentrate the pressure unduly (as indicated in Fig. 50) even if there is no danger of actual breaking of the journal. (b) If the product of the diameter times the length is too small the intensity of bearing pressure may expel the lubricant. The considerations involved in the design of journals and bearings are, therefore: I, Heating effect and wear. II, Intensity of bearing pressure. III, Strength or rigidity. IV, Energy lost through friction.

The rational procedure in determining the dimensions of a journal and bearing would seem to be about as follows:

(a) Determination of the necessary length from eq. (4), or eq. (5), Unwin, § 112

(b) Determination of the diameter necessary to give the proper intensity of bearing pressure with the length found in (a).

(c) Check of the journal with these values of d and l for strength or rigidity.

In these formulas for l, the value of P is to be taken as the mean load on the journal, as the heating is an effect of a continuous action. On the other hand, in checking for strength the maximum load must obviously be used. In the journals of engines and many other machines the maximum load is very often much in excess of the mean load. In line shafting, these two are practically equal in most cases.

The values of intensities of bearing pressures, such as are given by Unwin on page 198, should be considered as maximum values, *i. e.*, as about the maximum allowable values obtained in dividing the load by the product dl.

The proper formula for checking the strength or rigidity of a journal depends upon the bending moment, or other straining action on the journal; thus, for the strength of an overhung crank pin, like Fig. 137 (Unwin), the stress $= f = \frac{16}{\pi}\frac{Pl}{d^3}$. In the gen-

eral case of a journal subjected to a bending action alone, the stress is $f = \frac{32}{\pi}\frac{M}{d^3}$; in which M = the bending moment. If there is a combined bending and twisting action, the value of M in this expression should be replaced by that of the *equivalent bending moment*, (see Art. 16—Notes).

Unfortunately, the design of journals by the process just outlined is not wholly satisfactory; because the proper values of the constants β, or γ, of eq. (4), or (5), [Unwin, § 112], are not definitely determined for practical cases. These constants vary so widely with apparently small changes of the conditions which govern lubrication that it is not safe to infer their values except for cases very similar to those upon which observations have been made. An examination of several Corliss engines, each of which had a value of (H. P.) $\div R = 13$, showed values of γ ranging from .32 to .81; or crank-pin lengths ranging from 4.25″ to 10.5″. The tables of Unwin, on page 193, give, for rape oil, $\beta = 916{,}000$ with oil bath lubrication, while $\beta = 310{,}000$ for "siphon" feed. Professor Unwin says (§ 114), these values "must be divided by a factor of safety." On the other hand, higher values than those obtained by Mr. Tower (with his effective oil bath) are met with in practical operation when the pressure is intermittent, notwithstanding the inferior methods of lubrication in these latter cases.

While this theoretical discussion of the length of journals is important in giving a clear conception of the general effects of changes in relative proportions of length to diameter, it does not seem to afford an adequate basis for assigning actual dimensions for given cases. The design of a journal must, at best, depend largely upon judgment; first, because of the uncertainty and delicacy of the element of lubrication; second, because the journal must be given such dimensions as will insure not only satisfactory running under usual service conditions, but a fair degree of insurance against the contingencies that will probably arise during the life of the machine.

The intensity of bearing pressure (p) which can be allowed on a given class of journals, with velocities of rubbing and the char-

acter of lubrication usual in such class, is more definitely determined than are the coefficients β and γ. The allowable working stress for the common materials under known conditions may also be quite definitely assigned. These quantities, together with ratios of length to diameter which have been found generally satisfactory in practice, probably afford the most reliable data for design of journals.

In journals under very heavy load and running at very slow speed, strength is the primary consideration, intensity of bearing pressure comes second, and heating may need but little attention. In crank-pins of punching machines, for example, a short pin of large diameter gives the best combination of strength and bearing area. At higher speeds the length should be relatively greater; and in very high speed journals under small straining action much greater relative lengths are appropriate. These points can perhaps be best developed in the following articles, which treat some of the more common classes of journals separately.

73. Main Bearings and Crank-Pins of Punching and Shearing Machines.—In these machines, and others of a similar class, the rotative speed is usually low. Though the load on the crank-pin and main bearing of the shaft is great at its maximum, this load is only applied for a fraction of a revolution. These conditions are favorable to a high intensity of bearing pressure. The danger from over-heating is slight, hence the length of the bearing can be relatively small. If the maximum load, P, is assumed to be uniformly distributed along the crank-pin, the bending moment at the inner end of the pin (where the moment is greatest, with an overhung pin) is $\frac{1}{2} Pl$. (See Fig. 53).

The moment of resistance of the pin is $\frac{\pi}{32} d^3 f$.

$$\therefore\ d^3 = \frac{5.1\, Pl}{f} \qquad (1)$$

The intensity of bearing pressure is

$$p = \frac{P}{dl} \quad \therefore \quad dl = \frac{P}{p} \qquad (2)$$

From eqs. (1) and (2):

$$d^4 = \frac{5.1\,P}{f}\left(d\,l\right) = \frac{5.1}{f}\;\frac{P^2}{p}$$

$$\therefore\quad d = \sqrt[4]{\frac{5.1}{f\,p}}\;\sqrt{P} \qquad (3)$$

Having found d, its value can be substituted in the following expression to determine l:

$$l = \frac{P}{p\,d} \qquad (4)$$

Should the value of l be greater than seems desirable, it can be reduced by increasing d to maintain the required bearing area. Such a change will evidently reduce the stress in the pin.

In the table of "Pressures on Bearings and Slides" [Unwin, page 198], the allowable value of p is given at 3000 lbs. per sq. inch of projected area for such bearings as those treated in this article.

Example:—If $P = 70{,}000$ lbs.; $p = 3{,}000$; $f = 9{,}000$.

$$d = \sqrt[4]{\frac{5.1}{9{,}000 \times 3{,}000}}\;\sqrt{70{,}000} = 5.5''$$

Therefore, $$l = \frac{70{,}000}{3{,}000 \times 5.5} = 4.24''$$

To design the main journal of the shaft, the bending action and the load at this bearing should be known. The distance between the bearings and the "overhang" of the pin must be known to determine these (see Fig. 53). These dimensions would be fixed by the length of the pin and the design of the frame, and are readily determined for any given case.

The values of the reactions R_1 and R_2, and the bending moment at any section of the shaft, can readily be found by either mathematical or graphical means, when P and the above mentioned dimensions are known. If it is assumed that the reaction R_1 acts at the middle of the main bearing, the bending moment is a maximum at the point where R_1 acts, and this moment is $M = Pa$. The twisting moment on the shaft is $T = Pr$, when r is the throw

of the crank, or one-half the stroke of the machine. The bending moment equivalent to the combined bending and twisting actions is

$$M_e = \tfrac{1}{2} M + \tfrac{1}{2} \sqrt{M^2 + T^2} \qquad (5)$$

The assumption of $M = Pa$ is rather extreme, as the bearing affords a quite rigid support nearer the crank-pin than the middle of the bearing. The reaction and the moment at the back end of the shaft is much less than that at the main bearing, hence the shaft may be much smaller at the back end.

Equating M_e to the moment of resistance of the shaft, and solving for d:

$$d = \sqrt[3]{\frac{10.2\, M_e}{f}} \qquad (6)$$

Example :—If $P = 70{,}000$, $p = 3{,}000$, $f = 9{,}000$, as in the example of the crank-pin ; and $a = 6''$, $p = 30''$, $r = 1.5''$,

$$R_2 = \frac{70{,}000 \times 6}{30} = 14{,}000 \text{ lbs.}$$

$$R_1 = P + R_2 = 84{,}000 \text{ lbs.}$$

$$T = 70{,}000 \times 1.5 = 105{,}000 \text{ inch lbs.}$$

$$M = 70{,}000 \times 6 = 420{,}000 \text{ inch lbs.}$$

$$\therefore \quad M_e = 426{,}500.$$

$$\therefore \quad d = \sqrt[3]{\frac{10.2 \times 426{,}000}{9{,}000}} = 7.85''$$

The intensity of pressure on this bearing would usually be much less than that on the pin ; for, as illustrated in the above examples, the diameter of the main journal is increased over that of the crank-pin much more than the total load on this journal is increased over that of the pin, and the main journal would also be the longer. In this example, a length of main journal of only $3\frac{3}{4}$ inches would give an intensity of bearing pressure of 3,000 lbs. per sq. inch of projected area ; but such a length would be absurdly small. If the length of this journal is made 7 inches, the intensity of bearing pressure would be less than 1,600 lbs. per square inch.

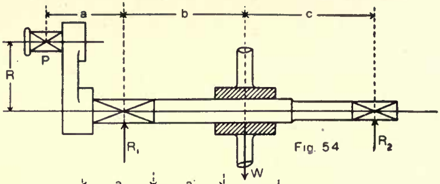


Fig. 54

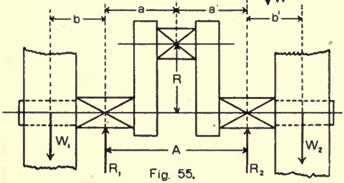


Fig. 55.

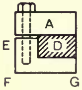


Fig. 57

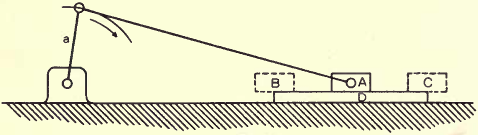


Fig. 56.

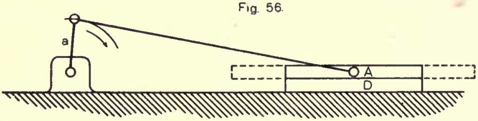


Fig. 58.

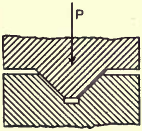


Fig. 60.

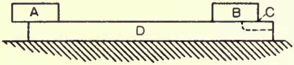


Fig. 59.

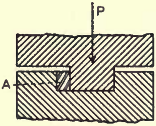


Fig. 61.

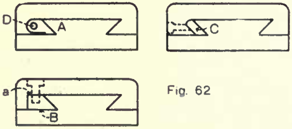


Fig. 62

74. **Engine Crank-Pins ; Side Crank.** [Unwin, §§ 115, 116, 118, 119, 120, 121.] The loads on engine crank-pins are considerable, though not usually so excessive as in punching and shearing machines; while the velocity of rubbing is so high that liability to heating cannot be neglected.

The length of the pin is a more important consideration than in the classes of journals treated in the preceding article. This length could first be determined by eq. (4) or eq. (5). [Unwin, § 112], except for the uncertainty as to proper values of β or γ, as referred to before.

A few years ago the principal dimensions of a large number of standard commercial engines were collected for use in Sibley College. Upon working this data up, the engines with overhanging cranks were separated from those with centre cranks. The dimensions of practice, as represented by about seventy-five side crank engines (mostly of the Corliss type), are expressed more closely by the formula :

$$l = c\frac{H.P.}{R} + K. \qquad (1)$$

than by the theoretical formula,

$$l = \gamma\frac{H.P.}{R}$$

as developed by Unwin [eq (5), § 112].

In the above equation (1), K has a value of about 2″, so that the minimum length by this formula would be more than 2″. The value of c ranges from 0.2 to 0.4 ; the mean value being about 0.3. It may then be said that, for engines of this class, viz : those with rotative speeds not over about 130 r. p. m. and from 50 to 600 H. P. capacity, the average length of crank-pin is about

$$l = 0.3\frac{H.P.}{R} + 2''. \qquad (2)$$

However, the value of c varies so greatly in the practice observed that it is not a satisfactory basis for design, and the following method seems preferable.

With side crank engines (overhanging pins) the ratio of the length to the diameter of the pins (λ) is usually from 1 to $1\frac{1}{4}$; the length being properly somewhat greater, relatively, at the higher rotative speeds. Unwin gives the relation,

$$\lambda = \frac{l}{d} = c\sqrt{N},$$

as desirable for journals in general; but this rule cannot be applied to overhung crank pins. With a uniformly distributed load on an overhung crank pin,

$$\tfrac{1}{2} P l = \frac{\pi}{32} d^3 f \quad \therefore \quad d^3 = \frac{5.1}{f} P l$$

$$\therefore \quad d^2 = \frac{5.1}{f} P \frac{l}{d} = \frac{5.1}{f} P \lambda . \tag{3}$$

But if p is the intensity of bearing pressure per square inch, $p\,(d\,l) = P$,

$$\therefore \quad p\, d^2\, l = P d \quad \therefore \quad d^2 = \frac{P d}{p\, l} = \frac{P}{p\, \lambda}. \tag{4}$$

Equating (3) and (4) and solving for λ:

$$\frac{5.1}{f} P \lambda = \frac{P}{p \lambda}, \quad \therefore \quad \lambda = \sqrt{\frac{f}{5.1\, p}} \tag{5}$$

From eq. (5) the value of λ corresponding to any assigned values of f and p can be found. If λ is taken at a lower value than that given by this last formula, the intensity of stress will be less than that assumed, and the pin will presumably be safe. If $f = 9000$ and $p = 1000$, $\lambda = 1\frac{1}{3}$. It thus appears that the limits of λ as given above (1 to $1\frac{1}{4}$) are safe for wrought iron or steel pins with about the customary maximum intensity of crank-pin bearing pressure.

From the relations of eq. (5) it is found that $f = 5.1\, p\, \lambda^2$. (6)

Substituting $5.1\, p\, \lambda^2$ for f, in eq. (3), and solving for d,

$$d = \sqrt{\frac{5.1\, P \lambda}{5.1\, p\, \lambda^2}} = \sqrt{\frac{P}{p\, \lambda}}. \tag{7}$$

The dimensions of the crank-pin may then be found as follows:

(a) Find the value of λ, from eq. (5), corresponding to proper values of f and p.

(b) With a value of λ *not exceeding* that determined as in (a), find d from eq. (7).

(c) Determine l from the relation,

$$\frac{l}{d} = \lambda, \quad \therefore \quad l = \lambda\, d. \tag{8}$$

Example: An engine 16″ × 20″, runs under a boiler pressure of 120 lbs. per sq, inch (gauge pressure). If f can be 8,000 lbs. per sq, inch, and $p = 1{,}000$, determine the proper dimensions for the overhung crank-pin. Assuming that the engine may be run condensing, the maximum unbalanced pressure on the piston may be about 132 lbs. per sq. inch.. This corresponds to a total load (P) on the piston of about 26,500 lbs.; say 27,000 lbs.

$$\lambda = \sqrt{\frac{8{,}000}{5\ 1 \times 1000}} = 1.25. \quad \text{Taking } \lambda = 1\tfrac{1}{8}, \text{ from eq. (7)}$$

$$d = \sqrt{\frac{27{,}000}{1000 \times 1.125}} = 4.9''.$$

$l = \frac{9}{8} \times 4.9 = 5.52.$ The diameter may be taken at 5″, and the length at $5\frac{1}{2}''$.

75. Crank Shaft Journals. Side Cranks. [Unwin, § 126.] The strength of shafts will be treated more fully in the next chapter, but the method of determining the journal dimensions will be outlined in this article. To determine the load at the main journals, the resultant reaction, R_1, (Fig. 54) must be known. When the distances a, b, c, and the direction and magnitude of the load on the crank pin, the total belt pull, and weight of fly wheel are known, complete data is available for computing the straining actions on the shaft and the dimensions of the journals. The distance a will often be about 4 to $4\frac{1}{2}$ times the diameter of the crank pin; and

it may be so assumed for preliminary computations, in absence of more definite data. The bending moment at the centre of the main journal, Fig. 54, (assuming the load as uniformly distributed along its length) is $M = Pa$; the twisting moment is $T = PR$; and the equivalent bending moment can be found as in art. 73, by eq. (5). The diameter of the shaft can then be found by eq. (6) of art. 73. The length (l) of the shaft main journal is often about twice its diameter but the projected area (ld) should be sufficient to keep the intensity of bearing pressure (p) within safe limits. Unwin gives p at from 150 to to 250 lbs. per sq. inch. The total load on the main journal is R_1 (Fig. 54), which is considerably greater than the load on the crank pin due to the steam pressure alone. When the total reaction R_1 cannot be determined, the projected area of the main journal may be taken, for ordinary power engines, as about the maximum load on the piston divided by 175. This intensity of pressure is much less than that allowed for the crank pin, because the component of the load on the shaft bearings due to fly wheel weight and belt pull is not intermittent like that due to the steam pressure.

If the required projected area makes the length of bearing such that the distance a is much greater than that assumed, it may be necessary to revise the computations.

The length of the crank hub (marked B in Fig. 54) is usually somewhat less than the diameter of the shaft.

The "outboard" bearing carries a much smaller load (R_2) than the main bearing; and when the power is delivered by the fly wheel, that portion of the shaft between the wheel and this outboard bearing is not subjected to torsion. Hence the latter bearing may be much smaller than the main bearing.

76. Centre Crank Shafts and Pins. Centre cranks, which have main bearings each side of the crank, are used on many high speed engines, on marine engines, and in other cases. In single crank stationary engines with this type of shaft there are frequently two fly wheels, one at each end of the shaft. Sometimes there are belts on both wheels, when part of the power is given off

by each wheel; sometimes one wheel delivers all of the power; and in other cases the power is delivered through direct connection with one or both ends of the shaft, the wheels acting purely as fly wheels. The weights of the wheels, the total belt pulls, the pressure on the piston, and the distances *a*, *b*, etc., must be known to completely determine the straining action at any section of the shaft and the load on any bearing. When the full data is at hand, the combined bending and twisting actions at the critical sections can be found, and the necessary diameters of such sections for strength can then be computed by methods similar to those of preceding sections; that is by placing $M_e = \frac{\pi}{32} d^3 f$, and solving for d.

The crank pin for a shaft such as that of Fig. 55 must be much larger than an overhung pin under a similar load; for this centre crank pin is really a portion of a beam supported at R_1 and R_2 and loaded at P, W_1 and W_2 The stresses are frequently severe at the junctions of the pin and of the two outside portions of the shaft with the crank arms, and liberal "fillets" should be provided at these angles. The analysis of the straining actions on such a shaft will be treated more fully in the next chapter.

It is quite common practice to give the pin of an engine which has a centre crank a diameter equal to that of the shaft at the main bearings. Some engine builders make the pin even larger than the shaft, while a few of them make it somewhat smaller. For the ordinary case it will be well to make the diameter of the pin as large as that of the main journals.

In the absence of any more definite data, the following method may be used for approximating the dimensions of the crank pin and main journals. Let $P =$ the maximum load on the crank pin, due to the steam pressure on the piston; H.P. $=$ the horse power of the engine at rated load; $N =$ the revolutions per min.; $d =$ the diameter of the shaft $=$ diameter of the crank pin. The diameter of the shaft should be

$$d = C \sqrt[3]{\frac{H.P.}{N}} \qquad (1)$$

Unwin gives (page 225) the value of C in eq. (1) as 4.55 for marine engines. For stationary high speed engines up to 250 or 300 H.P. C is usually between 6.5 and 8.5; the general average of practice corresponding to a value of $C = 7.3$, (about). The approximate diameter may be found by eq. (1), using a proper value of C.

When the diameter of crank pin has been fixed, the length can be found from the allowable intensity of bearing pressure (p)

$$p\,d\,l = P; \ \therefore l = \frac{P}{p\,d}. \tag{2}$$

The average value of p may be taken at about 450 lbs. per sq. inch. This is less than half the intensity of pressure frequently carried with overhung pins. The difference is accounted for as follows: The length of pin is the element which most affects the tendency to heat, as shown by Unwin, § 112; hence this length should be independent of the diameter. But the diameter of pin, necessary for strength, is much greater with the centre crank type, therefore the bearing area ($d\,l$) would be correspondingly greater with a proper length, or the intensity of pressure would be correspondingly less than with an overhung pin.

The intensity of bearing pressure on the main journals, due to the steam pressure on the piston alone, is often only from 100 to 125 lbs. per sq. inch. Assuming one-half of P to be carried by each of the main journals, the length of each is found from the following:

$$p\,d\,l = \tfrac{1}{2}P; \ \therefore l = \frac{P}{2\,p\,d}. \tag{3}$$

In which d has the value determined by eq. (1), above.

The distance from centre to centre of main bearings ($a + a' = A$, of Fig. 55) may be roughly determined thus:

$$A = \frac{P}{K\,d}. \tag{4}$$

An examination of a considerable number of standard engines shows that the value of K ranges from about 80 to 110, and a fair average value for use in preliminary computations is about 90.

This value is for solid forged shafts. If the shaft is "built up" by forcing the shaft and pin into crank arms or discs, the span A would usually be greater. The assumption that the shaft is a beam supported at the centre of the bearings is in the nature of an error on the safe side, as the effective span is probably considerably less, owing to the rigidity of the bearings.

The preceding method is only to be considered as approximate, yet it will perhaps meet the requirements in many cases. When the design has advanced sufficiently to furnish the full data, the dimensions assigned by the foregoing procedure should be checked.

77. Line Shaft Bearings The bearings of lines of shafting for transmitting and distributing power are generally so supported that they can swivel to a certain extent to accommodate the bearing surface to springing of the shaft. See Fig. 52 and Fig. 167 (Unwin). This freedom of the bearing avoids, to a very considerable degree, concentration of pressure at the edge of the bearing due to deflection of the shaft, such as is indicated by Fig. 51, and the bearing can be longer than would be practicable with rigid boxes or pillow-blocks. It is quite common practice to make the length of these line shaft swivel bearings about 4 times the diameter. The diameter of the bearing is the same as that of the shaft between bearings. These shafts are subjected to considerable bending, as well as to the torsion due to the power transmitted, when pulleys or gears delivering energy are at some distance from a bearing.

If the shaft is subjected to torsion only, the twisting moment is, as given in art. 11,

$$T = 63{,}024\,\frac{H.\,P.}{N} = \frac{\pi}{16}\,d^3 f. \qquad (1)$$

In which $H.\,P.$ = the horse power transmitted and N the revs. per min.

If the intensity of stress, f, is taken as a little less than 9,000 lbs. per sq. inch, the diameter of the shaft is

$$d = 3.3\,\sqrt[3]{\frac{H.\,P.}{N}}. \qquad (2)$$

When the bending action is so great that it must be considered, the equivalent twisting moment T_e should be used instead of T. See art. 16, eq. (4). In such cases the constant of eq. (2) is too small.

If the span between bearings does not exceed about 40 diameters, and if there be no heavy belt pull or gear thrust far from a bearing, the diameter may be taken as

$$d = 4.30 \sqrt[3]{\frac{H.P.}{N}}. \qquad (3)$$

The constant in eq. (3) is suitable for ordinary shop distribution shafting, For "head shafts" or other cases where the bending action is severe, it will be well to make

$$d = 4.64 \sqrt[3]{\frac{H.P.}{N}}. \qquad (4)$$

78 Crosshead Pins The usual type of short crosshead pin is supported at both ends. The diameter necessary for securing the bearing area (with such lengths as are generally convenient) gives excess of strength when the full sized pin is fitted into the crosshead. If, as is sometimes the case in the familiar "spade-handle" type of crosshead, the crosshead pin is a bushing held in place by a smaller pin passing through it, this smaller pin should be checked for resistance to shearing; but such a construction can be easily avoided. Occasionally the crosshead pin is of such form that the bending action must be considered. In this case it should be checked for strength as a beam

The pin can be designed, in the ordinary case, simply with reference to the allowable intensity of bearing pressure.

$$p\,d\,l = P, \therefore d\,l = P \div p. \qquad (1)$$

The length is commonly from 1 to $1\frac{1}{2}$ times the diameter; but as the velocity of rubbing is small, length of pin is not of first importance, and this ratio can be varied as dictated by considerations of the general design of the crosshead.

79. Bearings of Machine Tools, etc. [Unwin, § 117.] In most machine tools, and in many other machines, the loads upon the journals are comparatively small, and are too indefinite to become the basis for satisfactory numerical computations. The requirements of rigidity and permanence of form (minimum wear) often lead to the adoption of journals much larger than any considerations of strength would dictate. Liberal bearing areas are the rule in such classes of machines. As the speed increases the length of bearing should increase, if other considerations permit. A general guide to determination of length may be found in § 117 of Unwin.

80. Thrust Bearings. [Unwin, §§ 130–131.] The intensity of pressure (p) on a thrust bearing may be computed by the following formula, when the velocity of rubbing (v) is from 30 to 170 feet per minute at the outer edge of the step,

$$p = \frac{7500}{v - 20}. \qquad (1)$$

This expression gives an intensity of 750 lbs. at a velocity of 30 feet per minute, and of 50 lbs. at 170 feet per minute: which may be taken as about the limits of pressure for steps or thrust bearings.

Except with low rubbing velocities, the pressures should be quite moderate; because, unlike ordinary journals, the wear on different portions of the contact surfaces is very different. The rubbing velocity is zero at the centre, and it increases with the radius to a maximum at the outer edge. Hence the tendency to wear is greater at the outer part. Such wear concentrates the pressure near the centre, on a smaller area, than the nominal bearing surface. The effect of this action is to increase the real intensity of pressure and induce cutting. A large surface and a low intensity of pressure reduces the rate of wear; but a more effective means of securing durability is to use the collar-thrust bearing, as shown in Fig. 139 (Unwin). With such a collar bearing, the difference between maximum and minimum velocities of rubbing depends upon the ratio of the outer diameter of collar (d_1) to the inner diameter (d_2). To keep this difference as

small as possible and also to reduce the frictional work to a minimum by avoiding an unnecessarily high velocity, the outer diameter should not be very much larger than the shaft diameter. The required surface can be supplied by increasing the number of rings. Of course practical considerations limit the increase in number and the decrease in size of the collars.

81. Construction of Bearings, Pedestals, etc. [Unwin, Chapter VIII, pages 244 to 268.] The almost universal practice in this country is to use the swivel form of bearing for line shafting, Fig. 167 (Unwin), or its equivalent. This is used not only with pillow blocks, as shown, but with post-hangers (Unwin, Fig. 170), and drop-hangers (Unwin, Fig. 172). The bearing is usually of cast iron without any lining material, as this metal is satisfactory under the low intensity of bearing pressure possible with these long swivel bearings.

For rigid bearings, Babbitt metal, or some other so called "anti friction" alloy, is quite commonly used as a lining. For many purposes this lining is simply cast in the box around the shaft or around an arbor. If cast on the shaft with which it is to be used, the shaft should be wrapped with paper, which gives clearance in the bearing and also reduces the danger of springing the shaft by the heat. If the shaft is of cold rolled steel, the danger of springing it is considerable. In all such cases it is preferable to cast the lining around a special arbor of slightly larger diameter than that of the shaft to be run in the bearing.

The shrinkage of the lining metal, in cooling, is apt to leave it somewhat loose in the supporting shell. For the classes of work requiring greater accuracy, or where the pressures on the bearing are apt to be severe, the proper procedure is to cast the lining on an arbor considerably smaller than the shaft; then to compress or "pene" the metal after it has cooled, and finally bore it out to the required size. This compression of the soft metal causes it to fill the containing cavity snugly, correcting the effect of shrinkage; it also gives a firmer material and one less liable to be "hammered" out of shape under service load.

Professor Unwin says (page 250) the end play of shafts may be one-tenth the length of journal. This is an extreme amount,

and the collars would seldom be set to allow as much end play in bearings of ordinary dimensions.

In long lines of shafting the expansion or contraction with changes of temperature may be considerable. For this reason the collars for longitudinal constrainment should be placed near the point at which it is desired to have the least axial motion. Thus, if there should be a bevel gear along the line, the collars should be placed at the nearest bearing to such gear. If there are no gears, however, the collars may be placed near the middle of the line, or at any point where there is special reason for maintaining the longitudinal position of a pulley or clutch.

82. Rectilinear Sliding Surfaces. In the design of slides and guides for securing relative translation between two members, the bearing area should be taken with reference to the intensity of bearing pressure. This intensity should be less as the velocity of rubbing becomes greater. The values given by Professor Unwin in § 121 may be used for engine crosshead shoes; though with small high speed engines the intensity of bearing pressure is often not more than 20 to 30 lbs. per sq. inch.

The bearing surfaces of the slides of machine tools should be liberal. They may well be as large as it is convenient to make them; for comparatively slight wear impairs the accuracy of the output of such machines.

The following discussion of slides is taken, by permission, from the work on Machine Design by Professor Albert W. Smith, of Leland Stanford University:

"So much of the accuracy of action of machines depends on the sliding surfaces, that their design deserves the most careful attention. The perfection of the cross-sectional outline of the cylindrical or conical forms produced in lathes, depends on the perfection of form of the spindle. But the perfection of the outlines of a section through the axis depends on the accuracy of the sliding surfaces. All of the surfaces produced by planers, and most of those produced by milling machines, are dependent for accuracy on the sliding surfaces in the machine.

Suppose that the short block *A*, Fig. 56, is the slider of the slider-crank chain, and that it slides on a relatively long guide,

D. The direction of rotation of the crank, A, is as indicated by the arrow. B and C are the extreme positions of the slider. The pressure between the slider and the guide is greatest at the mid-position, A, and at the extreme positions, B and C, it is only the pressure due to the weight of the slider. Also the velocity is a maximum when the slider is in its mid-position, and decreases toward the ends, becoming zero when the crank, A, is on its center. The work of friction is therefore greatest at the middle, and is very small near the ends. Therefore the wear would be greatest at the middle, and the guide would wear concave. If now the accuracy of a machine's working depends on the perfection of A's rectilinear motion, that accuracy will be destroyed as the guide, D, wears. Suppose a g b, EFG, to be attached to A, Fig. 57, and to engage with D, as shown, to prevent vertical looseness between A and D. If this gib be taken up to compensate wear after it occurred, it would be loose in the middle position when it is tight at the ends, because of the unequal wear. Suppose that A and D are made of equal length, as in Fig. 58. Then when A is in the mid-position corresponding to maximum pressure, velocity, and wear, it is in contact with D throughout its entire surface, and the wear is therefore the same in all parts of the surface. The slider retains its accuracy of rectilinear motion regardless of the amount of wear, the gib may be set up, and will be equally tight at all positions.

If A and B, Fig. 59, are the extreme positions of a slider, D being the guide, a shoulder would be finally worn at C. It would be better to cut away the material of the guide, as shown by the dotted line. Slides should always "wipe over" the ends of the guide when it is possible. Sometimes it is necessary to vary the length of stroke of a slider, and also to change its position relatively to the guide. Examples: "Cutter bars" of slotting and shaping machines. In some of these positions, therefore, there will be a tendency to wear shoulders in the guide and also in the cutter bar itself. This difficulty is overcome if the slide and guide are made of equal length, and the design is such that when it is necessary to change the position of the

cutter bar that is attached to the slide, the position of the guide may be also changed so that the relative position of slide and guide remains the same. The slider surface will then just completely cover the surface of the guide in the mid-position, and the slider will wipe over each end of the guide, whatever the length of the stroke.

In many cases it is impossible to make the slider and guide of equal length. Thus a lathe carriage cannot be as long as the bed ; a planer table cannot be as long as the planer bed, nor a planer saddle as long as the cross-head. When these conditions exist special care should be given to the following ; I. The bearing surface should be made so large in proportion to the pressure to be sustained that the maintenance of lubrication shall be insured under all conditions. II. The parts which carry the wearing surfaces should be made so rigid that there shall be no possibility of the localization of pressure from yielding.

As to form, guides may be divided into two classes : angular guides and flat guides. Fig. 60 shows an angular guide, the pressure being applied as shown. The advantage of this form is, that as the rubbing surfaces wear, the slide follows down and takes up both the vertical and lateral wear. The objection to this form is that the pressure is not applied at right angles to the wearing surfaces, as it is in the flat guide shown in 61. But in Fig. 61 a gib, *A*, must be provided to take up the lateral wear. The gib is either a wedge or a strip with parallel sides backed up by screws.

Guides of the angular forms are used for planer tables. The weight of the table itself holds the surfaces in contact, and if the table is light the tendency of a heavy side cut would be to force the table up one of the angular surfaces away from the other. If the table is very heavy, however, there is little danger of this, and hence the angular guides of large planers are much flatter than those of smaller ones. In some cases one of the guides of a planer table is angular and the other flat. The side bearings of the flat guide may be omitted, as the lateral wear is taken up by the angular guide. This arrangement is undoubtedly good if both guides wear down equally fast.

Fig. 62 shows three forms of sliding surfaces such as are used for the cross slide of lathes, vertical side of shapers, the table slide of milling machines, etc. *A* is a taper gib that is forced in by a screw at *D* to take up wear. When it is necessary to take up wear at *B*, the screw may be loosened and a shim or liner may be removed from between the surface at *a*. *C* is a thin gib, and the wear is taken up by means of several screws like the one shown. This form is not so satisfactory as the wedge gib, as the bearing is chiefly under the points of the screws, the gib being thin and yielding, whereas in the wedge there is complete contact between the metallic surfaces.

The sliding surfaces thus far considered have to be designed so that there will be no lost motion while they are moving, because they are required to move while the machine is in operation. The gibs have to be carefully designed and accurately set so that the moving part shall be just "tight and loose," *i. e.*, so that it shall be free to move, without lost motion to interfere with the accurate action of the machine. There is, however, another class of sliding parts, like the sliding head of a drill press, or the tail stock of a lathe, that are never required to move while the machine is in operation. It is only required that they shall be capable of being fastened accurately in a required position, their movement being simply to readjust them to other conditions of work, while the machine is at rest. No gib is necessary and no accuracy of motion is required. It is simply necessary to insure that their position is accurate when they are clamped for the special work to be done."

VIII.

AXLES, SHAFTING. AND COUPLINGS.

83. Definitions and General Equations. [Unwin, § 133, pages 208, 209]

84. Axles loaded transversely. [Unwin, §§ 134, 135, 136.]

85. Shafts under Torsion only. [Unwin, §§ 137, 138]

86. Shafts subjected to combined Torsion and Bending. [Unwin, § 139.] A convenient diagram is shown in Fig. 63 for determining the diameter of a shaft, of solid circular cross-section, subjected to any moment, and with any intensity of fibre stress from zero to 15,000 lbs. per sq. inch. This diagram can be used for either simple bending or twisting moments, or for combined bending and twisting actions. Its use in connection with problems involving simple twisting moments will be discussed first.

If T is the twisting moment, d the diameter of the solid circular shaft, and f the intensity of stress in the most strained fibres, $T=\frac{\pi}{16}fd^3$. Therefore, for a given diameter of shaft, T is directly proportional to f. Thus, if $d=4''$, $d^3=64$, and $T=.196\times 64f=12.57f$. If f be taken as 10,000, $T=125,700$ inch lbs. In Fig. 63, if ordinates represent moments (to the scale "A," of 10,000 inch lbs. to the 1"); and if abscissas represent intensity of stress (to the upper scale, "B," of 4,000 lbs. per sq. inch to the inch), the point a corresponds to $T=125,700$, $f=10,000$, $d=4''$. As the moment varies directly as the intensity of stress, for any given diameter of shaft, the relations between corresponding values of T and f (for a 4" shaft) will be represented by the straight line through the point a, and the origin O. In a similar manner straight lines through the origin are drawn for other shaft diameters.

To determine the diameter of shaft for a moment of 90,000 inch lbs, with a fibre stress of 12,000 lbs. per sq. inch, pass along the horizontal through the point marked "9" (or $T = 90{,}000$) on scale "A," to the vertical line through the point marked "12" (or $f = 12{,}000$) on scale "B." The intersection of this horizontal and vertical (b) lies a little below the diagonal marked 3.4 at its outer end; or the shaft should be about $3.37''$ or $3\frac{3}{8}''$ diameter to give a stress of 12,000 per sq. inch.

The oblique line nearest to the point located in the last example bears three figures, viz.: ".738—1.58—3.4," and the other diagonals each bear three separate figures. The significance of these designations will be explained by further illustrations.

If $T = \frac{1}{10}$th of 90,000, or 9,000 inch lbs. and $f = 12{,}000$,

$$d = \sqrt[3]{\frac{16\,T}{\pi f}} = \sqrt[3]{\frac{16}{12{,}000\,\pi}} \sqrt[3]{\frac{90{,}000}{10}} = 3.37 \div \sqrt[3]{10} = 1.56'' + ;$$

since d varies as the cube root of T and when $T = 90{,}000$, $d = 3.37''$.

In a similar way, if $T = 900$, or $\frac{1}{100}$th of 90.000, $d = 3.37 \sqrt[3]{100} = .726''$.

To use the diagram when $T = 900$, and $f = 12{,}000$, consider scale "A" as representing the moment in 100 inch lbs.; pass along the horizontal through 9 of this scale to the vertical through 12 of scale "B," as before, to the point "b," and take the *first* figure borne by the nearest diagonal (.732) as the approximate diameter of the shaft; or, by interpolation, find the diameter $= .724''$.

If $T = 9{,}000$, $f = 12{,}000$; consider scale "A" as representing the moment in 1,000 inch lbs., and read the *middle* figure on the nearest diagonal (1.58) as the required approximate diameter of the shaft; or, by interpolation, the diameter is found to be $1.56''$.

If the moment is greater than 130,000 (and less than 1,300,000) the diagram is quite as applicable as for smaller moments. Thus if $T = 900{,}000$ and $f = 12{,}000$, consider scale "A" as representing the moment in 100,000 inch lbs. The horizontal through 9 of scale "A" and the vertical through 12 of scale "B" intersect at "b" as before. The required diameter is about $7.24''$; because

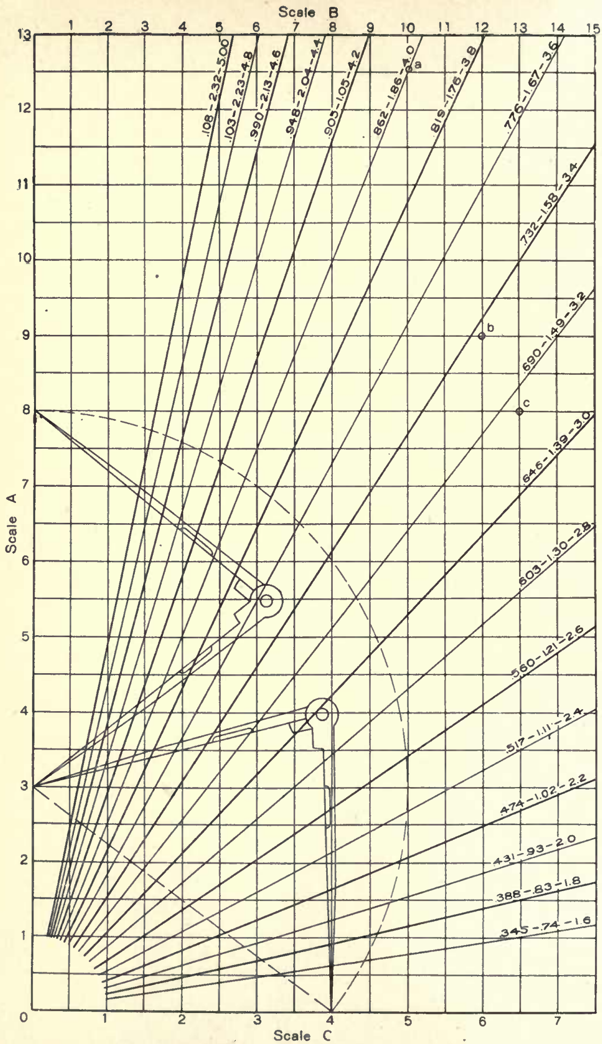

Scale B
Scale A
Scale C
.108-2.32-5.00
.103-2.23-4.8
.990-2.13-4.6
.948-2.04-4.4
.905-1.05-4.2
.862-1.86-4.0
.819-1.76-3.8
.776-1.67-3.6
.732-1.58-3.4
.690-1.49-3.2
.646-1.39-3.0
.603-1.30-2.8
.560-1.21-2.6
.517-1.11-2.4
.474-1.02-2.2
.431-.93-2.0
.388-.83-1.8
.345-.74-1.6
a
b
c

FIG. 63.

the diameter was found to be about .724 for a moment of 900, and it must be 10 times as great for a moment of $10^{-3} \times 900 = 900,000$. For $f = 12,000$ with a moment of 9,000,000 inch lbs. ($= 10^{-3} \times 9,000$), the diameter is $10 \times 1.57 = 15.7''$, etc. It thus appears that the diagram covers all moments, without being of such impracticable size as it would be if it were not for the peculiar designation of the oblique lines and the method of using scale "A." The diagram can also be used for simple bending moments. The expression for the bending moment in an axle of solid circular section is

$$M = \frac{\pi}{32} f d^3;$$

while the expression for a twisting moment is, as given above,

$$T = \frac{\pi}{16} f d^3.$$

Therefore, with a given diameter and numerically equal fibre stress, T is numerically equal to $2\,M$. To determine d for given values of f and M, multiply M by 2 to get the equivalent T, and with this value of T, proceed as in the former examples.

For finding the diameter appropriate to a combined bending and twisting moment, the equivalent twisting moment,

$$T_e = M + \sqrt{M^2 + T^2},$$

is to be determined; see art. 16. This equivalent twisting moment is readily determined from the diagram by use of scale "C" at the bottom of Fig. 63 and a pair of dividers, when the simple bending moment (M) and the simple twisting moment (T) are given. Example: Suppose $M = 30,000$, $T = 40,000$, and $f = 13,000$. Consider scales "A" and "C" to measure moments in 10,000 inch lbs. Take M at 3 *on scale "A"* with one point of the dividers, and T at 4 *on scale "C"* with the other point of the dividers; then the distance between 3 on scale "A" and 4 on scale "C" represents $\sqrt{M^2 + T^2}$. Swing the dividers about the point at 3 on scale "A" as a centre until the other point reaches scale "A" (at point 8); then 0 . . 8 on scale "A" = 0 . . 3 + 3 . . 8 = $M + \sqrt{M^2 + T^2} = T_e$. With the value of T_e, found in this way,

proceed as in case of a simple twisting moment. The intersection of the horizontal through 8 (T_e) and the vertical through 13 (f) is at point "c." Since the moments correspond to units of 10,000 inch lbs. on scale "A," the largest figures of the diagonals are to be read in determining the diameter. The point "c" therefore indicates a diameter of between 3.0″ and 3.2″; by interpolation the diameter is taken as 3.15″. By computation the diameter is found to be 3.14″. A shaft $3\frac{3}{16}$″ diameter would be proper for this case.

The diagram of Fig. 63 is equallv convenient for finding the intensity of stress in a given shaft under a known moment; or the moment on a given shaft corresponding to any intensity of stress. Thus, if a $7\frac{3}{4}$″ shaft is subjected to moment of 1,000,000 inch lbs., consider the moment units as 100,000 inch lbs., pass horizontally from 10 on scale "A" to a point slightly below the diagonal marked .776 (7.76″ diameter), and then vertically upward to scale "B," where the stress is read as about 10,950 lbs. per sq. inch.

If it is required to find the twisting moment corresponding to an intensity of stress of 9,000 lbs. per sq. inch on a shaft $1\frac{1}{2}$″ diameter; pass vertically downward from "9" on scale "B" to a point somewhat above the diagonal marked "1.49"; then horizontally to 5.9 on scale "A." As 1.49 is the middle number on the diagonal, the moment units are 1,000 inch lbs.; therefore $T = 5.9 \times 1{,}000 = 5{,}900$ inch lbs.

87. Mill Shafting. [Unwin, § 140.] See, also, eq. (2), (3) and (4) of article 77 (Notes).

88. Hollow Shafts. [Unwin, § 141.] The use of hollow shafts not only reduces the weight for a given strength, but the removal of the metal from the core of a steel shaft (or of the ingot from which it is made) very greatly increases its reliability under repeated application of stress.

Shortly after a steel ingot is cast, the exterior solidifies and becomes comparatively cool while the internal portion is still fluid. The subsequent contraction, during complete cooling, is much less in the exterior walls than it is in the hotter interior mass. Unless the interior is "fed" during this period, it will

be less dense than the outer portions and shrinkage cavities are apt to be present in the ingot. Numerous expedients have been adopted to reduce this evil, among which is "fluid compression," or subjecting the ingot to heavy pressure immediately after it is poured. The difficulty is not entirely overcome by such means, however, as the walls of large ingots become too rigid to yield to the pressure before the interior is entirely solidified. The external walls "freeze," after which the internal shrinkage is made up by metal flowing from the upper portion toward the bottom as long as any of it remains fluid. This leaves a shrinkage cavity at the upper end of the ingot. Gas liberated during cooling collects in this cavity also. The result of these two actions is to form what is called the "pipe," which frequently extends to a considerable depth. The top end of the ingot is cut off and remelted, but this does not insure removal of all of the pipe, and it involves much expense. If the portion cut off is not sufficient to remove all of the pipe, a piece rolled or forged from the ingot contains a flaw near the centre which is drawn out into a long crack if the ingot is worked into a long piece. The rolling or forging may squeeze the sides of the cavity together so that it is not easily detected at any section, but as this work is done at a temperature much below that corresponding to welding, the defect is not removed. This flaw is more or less irregular or ragged, hence its form is favorable to starting a fracture, under variations of stress, which may finally extend far enough to cause rupture. See the discussion of "micro-flaws" and gradual fractures on page 12.

If the ingot is bored out, the pipe is effectually removed, and the metal remaining is superior to that of a solid shaft. It will be evident that casting a hollow ingot is not the equivalent of boring out one which was cast solid; for if the ingot is cast hollow the outer and inner walls cool before the intermediate mass does, and the shrinkage effect takes place in the latter. In fact, a shaft made from a hollow ingot is worse than the solid shaft, in the respect that the former has the defective material nearer the outer fibres where the stress is greater.

89. Span, or Distance between Bearings, in Lines of Shafting. [Unwin, §§ 142, 145]. The deflection of a beam is proportional to the load upon it, to the cube of the span, and inversely as the moment of inertia of the section. With a shaft of solid circular section, the transverse load due to its own weight is proportional to the square of the diameter, and the moment of intertia is proportional to the fourth-power of the diameter. Hence, the deflection is proportional to $d^2 L^3 \div d^4$, or to $L^3 \div d^2$; and, for a given limit of deflection, $L = \gamma \sqrt[3]{d^2}$, which is eq. (41) of § 142 (Unwin).

Kent's Mechanical Engineers' Pocket-Book (page 868) says: "The torsional stress is inversely proportional to the velocity of rotation, while the bending stress will not be reduced in the same ratio. It is, therefore, impossible to write a formula covering the whole problem and sufficiently simple for practical application, but the following rules are correct within the range of velocities usual in practice. For continuous shafting so proportioned as to deflect not more than $\frac{1}{100}$ of an inch per foot of length, allowance being made for the weakening effect of key-seats,

$$d = \sqrt[3]{\frac{50\,H.P.}{R}},\quad L = \sqrt[3]{720\,d^2}, \text{ for bare shafts;}$$

$$d = \sqrt[3]{\frac{70\,H.P.}{R}},\quad L = \sqrt[3]{140\,d^2}, \text{ for shafts carrying pulleys, etc.}$$

d = diam. in inches, L = length in feet, R = revs. per min."

If the length of span is expressed in inches, as in eq. (41), Unwin, Kent's constants correspond to $\gamma = 108$ for bare shafts, and 62 for shafts with pulleys, etc.

It is well to check by the formula of § 145, Unwin, when the speeds are high.

90. Cold Rolled Shafting. [Unwin, § 143]. Shafting which is finished by a cold rolling process, instead of by turning it, is largely used. It is very true as to cross-sections. The cold working raises the elastic strength of the shaft; this effect being greatest near the outside, which is the portion subjected to the highest stress.

Cold rolled shafting is peculiarly liable to be "sprung" in cutting key-ways, etc., as this operation removes part of the most compressed metal, and thus disturbs the condition of internal stress in the material.

91. Expansion of Shafts. [Unwin, § 144].

92. Crank Shafts. [Unwin, §§ 146, 147, 148]. An analysis of the centre crank type of shaft, similar to that given by Professor Unwin (§ 148) for the side crank form, is outlined below. Fig. 64 shows a centre crank shaft of the form commonly used with high speed engines. It will be assumed that the two fly-wheel pulleys are of equal weight, and that the member is symmetrical about the centre of the crank pin; *i. e.*, that $a = a'$, $b = b'$. The forces on the shaft are: the load (P) on the crank pin due to the steam pressure on the piston and the inertia effects of reciprocating parts; the gravity action on the wheels, W_1 and W_2, the belt pulls, $T_2 + T_1$; the reaction at the main bearings, R_1 and R_2.

Graphical analysis, in conjunction with computations, is here very convenient. There are two cases to be considered: in the first the power is delivered by a belt on one fly-wheel; in the other the power is divided, part being delivered by each wheel. The former results in the more severe straining actions on the shaft, and it will therefore be taken for discussion, There is always one element of uncertainty in designing such a shaft for an engine not built for some particular service, viz.: the direction in which the belt will lead; but this is not of the first importance, and the condition shown in Fig. 64 represents about the maximum straining actions for a horizontal engine.

The force transmitted to the crank pin by the connecting rod, (P) varies in direction with the angularity of the rod, but it is sufficiently exact to consider this force as acting parallel to the centre line of the engine, or perpendicular to the plane of the paper in Fig. 64. The reaction at each main bearing due to P is $\frac{1}{2} P$. For a horizontal engine, this force is represented by $\frac{1}{2} P$ in Fig. 64 (a); the reaction at each bearing due to the fly wheel weight is $W_1 = W_2$, since the crank and wheels are assumed to be symmetrically disposed relatively to the bearings.

Let W represent the weight of one wheel in Fig. 64 a. Assum-

ing all of the power to be given off at one wheel (as W_1), the resultant pull from the belt on this wheel is $T_2 + T_1$, when $T_2 =$ total pull on the tight side and $T_1 =$ total pull on the slack side of the belt. If this resultant belt pull, be represented by $T_2 + T_1$ in Fig. 64 (a), the total reaction at the left hand bearing is equal and opposite to the resultant of the system of forces: $\frac{1}{2} P$, W, and $T_2 + T_1$. With the values of these forces and the direction of $T_2 + T_1$ assumed in Fig. 64 (a), the resultant force acting at the wheel (due to gravity and belt pull) is approximately horizontal; hence its line of action, and that of the resultant R', nearly coincides with that of the load on the crank pin. This condition gives the maximum straining action on the shaft, and is therefore a safe assumption. Assuming, then, that S and R_1 are in the same plane; take a section, $x\,x$, through the centre of the crank pin and compute the bending moments of all external forces to the left of this section with reference to it. If $m\,n$ (Fig. 64 b) is the bending moment of the force S at section $x\,x$, $m\,n\,g$ is the moment diagram of this force. The moment due to S at any section is given by that vertical ordinate of the diagram which is at a distance from g equal to the distance of the section from this same point. In a similar way, if $n\,q$ is the bending moment due to R_1, about $x\,x$, $n\,q\,h$ is the moment diagram for R_1. But the moments due to S and R_1 are of opposite signs, hence the diagram of unbalanced moments is the shaded area, $m\,q\,h\,g\,m$. The twisting moment on the shaft between the centre of the pin and the wheel is equal to Pr. Draw the rectangle $m\,i\,j\,g$ with a height mi representing this moment to the same scale used for the bending moments. Combine the unbalanced bending moments for various sections with the twisting moments (by the methods used in §§ 147, 148 of Unwin) and the diagram $k\,l\,s\,t$ is the diagram of the equivalent bending moment (M_e) on the left hand half of the shaft. This equivalent moment is seen to be a maximum at the centre of the main bearing, and the diameter of shaft should be computed for this maximum moment by the equation

$$d = \sqrt[3]{\frac{10.2\,M_e}{f}} \qquad (1)$$

The diameter at any other section may be computed by equation (1), using the value of M_e appropriate for that section. The shaft is made of the same diameter throughout its length, or it is reduced somewhat from the outer limits of the bearings to the ends. As previously stated, the crank pin is frequently made with a diameter equal to that of the main bearings.

It will be noticed from Fig. 64 b that the bending moment is zero at v. Inasmuch as fracture is particularly liable to start at the junctions of the pin or the shaft with the crank arms, it appears desirable to have this section of zero bending moment about mid-way between these two junctions, or at about the middle of the crank arm. In a vertical engine, the maximum straining action occurs when the belt pull is vertically downward. While this is not the most common condition, it is safest to assume it for the general case.

93. **Couplings.** [Unwin, §§ 149 to 155].

The standard coupling is the flange coupling shown by Figs. 149 and 150 (Unwin). Compression couplings are also much used in lines of shafting. The form shown in Fig. 151 (Unwin) is largely used in this country; as is also a simple clamp coupling similar to that of Fig. 152 (Unwin), excepting that the two halves are more often held together by bolts passing each side of the shaft instead of by the bands at the ends.

94. **Clutches.** [Unwin §§ 156, 157].

The prong, or jaw, clutch is often used for connecting sections of shafts which do not have to be frequently engaged or disengaged; or when this does not have to be done when either shaft is running.

The form of friction clutch shown in Fig. 155 (Unwin) is not uncommon, especially for engaging a loose pulley with a shaft; though the more usual form for friction cut-off couplings and clutch pulleys is one in which wooden faced jaws, actuated by a system of levers and toggles, clamp a ring or plate to engage the clutch.

95. **Universal Coupling.** [Unwin, § 158].

The Hooke's joint is a useful device in a limited way. The nature of the motion transmitted by this mechanism is discussed in Kinematics of Machinery.

IX.

FRICTIONAL AND TOOTHED GEARING.

95. **Frictional Gearing.** [Unwin, §§ 177, 178, 183]. See, also, Kinematics of Machinery, (Barr), arts. 51 to 55.

Professor Goss, of Purdue University, in a paper read before the A. S. M. E. (Trans., Vol. XVIII, p. 102), reported the results of some tests of friction wheels from which the following abstract is derived :

These experiments were made with driving wheels having friction surfaces of compressed straw board, and followers having turned iron faces.

This combination gives greater resistance to slipping than two metallic wheels. The softer material should always be used for the face of the driving wheel, in order that the wear resulting, should the follower stop under load, will be distributed around the circumference instead of being concentrated at one spot. The above mentioned experiments indicate that :

Slippage increases gradually with the load up to 3 per cent., but when the slippage is between the limits of 3 and 6 per cent. it is apt to suddenly "increase to 100 per cent. ; that is, the driven wheel is likely to stop."

The Coefficient of Friction is most affected by slippage. "Its value increases with increase of slip until the latter becomes about 3 per cent., after which the action of the gearing becomes uncertain. With a slippage of 2 per cent., the maximum value of the coefficient rises above 25 per cent." A value of 20 per cent. is easily attainable with wheels of 8 inches diameter and upward. The coefficient is apparently constant for all pressures of contact up to 150 to 200 lbs. per inch of width of face ; but it decreases with higher pressures. "Variations in peripheral speed between 400 and 2,800 feet per minute do not affect the coefficient of friction."

Pressure of Contact. The power transmitted varies directly with the pressure of contact; the coefficient of friction remaining constant. In the limited duration of the experiments, no indication of deterioration of the surfaces were noted under a pressure of 400 lbs. per inch of face; but the most efficient pressure is about 150 lbs. per inch of face.

"*Horse Power.* By making d the diameter of the friction wheel in inches, w the width of face also in inches, and N the revolutions per minute, and by accepting 0.2 as a safe value for the coefficient of friction, and a pressure of 150 pounds per inch of width of face as the pressure of contact, the horse-power may be written as:

$$H.P. = \frac{150 \times 0.2 \times \frac{1}{12} \pi d \times w \times N}{33{,}000} = .000238\, d\, w\, N.$$

This formula is believed to be safe for friction wheels which are eight inches or more in diameter, and under conditions which make it possible for them to be kept reasonably clean."

96. Wedge Gearing, or "V Frictions." [Unwin, § 184]. See, also, Kinematics of Machinery, art. 55.

If the total normal pressure on all the wedge surfaces be N, the force pressing the wheels together be P, and the tangential force transmitted be T,

$$N = P \div \sin \alpha \qquad (1)$$

in which α is half the angle between the adjacent faces of each groove (or wedge). The angle 2α is often about 40°.

$$T \overline{\overline{<}} \mu N, \overline{\overline{<}} \mu P \div \sin \alpha \qquad (2)$$

when μ is the coefficient of friction between the contact surfaces.

If the pitch diameter of the wheel (the diameter to the middle of the depths of the grooves) be D feet, and the revolutions per minute be n, the power transmitted is,

$$H.P. = \frac{T \pi D n}{33{,}000} \overline{\overline{<}} \frac{\mu P \pi D n}{33{,}000 \sin \alpha} \qquad (3)$$

$$\therefore \quad H.P. \overline{\overline{<}} \frac{\mu P D n}{10{,}500 \sin \alpha} \qquad (4)$$

$$\therefore \quad P \overline{\overline{>}} \frac{10{,}500 \sin \alpha \, H.P.}{\mu D n} \qquad (5)$$

Mr. Kent states, on page 906 of the "Pocket Book," that: "The value of μ for metal on metal may be taken at .15 to .20; for wood on metal, .25 to .30." The number of grooves, for a given angle α, does not effect the relation between P and T. But, for a given face of wheel, the depth of grooves is increased as the number is decreased, and the grinding action between adjacent surfaces is proportional to the depth of the contact faces of the "V's." See Kinematics of Machinery, page 106.

97. General Features of Toothed Gearing. [Unwin, §§ 185 to 190, inclusive].

The relations given by Professor Unwin in §§ 191 to 209, inclusive, are usually treated in courses on Kinematics, and are therefore omitted from these Notes.

98. Power Transmitted by Toothed Gearing. The power transmitted and the resulting straining actions on gears and their shafts will now be briefly treated. The following notation is used throughout this article. See Fig. 65.

$P' =$ the normal pressure on a tooth, which acts along the line connecting the pitch point of the gear (a) with the contact point (g).

$P =$ the tangential component of the preceding, or the pressure which acts tangentially to the pitch circle.

$R =$ the radius of the pitch circle.

$N =$ the revolutions per unit of time.

$T =$ the turning moment on the gear and shaft.

$M =$ the bending moment on the shaft.

If the foot-inch-minute system of units be taken, the turning moment in inch pounds is,

$$T = PR, \qquad (1)$$

and the energy transmitted in inch pounds per minute is,

$$E = 2\pi N \cdot PR, \qquad (2)$$

equal to the angular velocity times the rotative moment, or to the tangential pressure times the linear velocity.

$$2\pi NPR = 12 \times 33{,}000\ H.P.$$

$$\therefore \qquad P = 63{,}020 \frac{H.P.}{RN} \qquad (3)$$

The normal pressure, $P' = P \sec \theta$; but the arm of this force (P') about the axis of the shaft, is R', while the arm of the tangential force (P) is R; Fig. 65. Since $R = R' \sec \theta$, $T = P' R' = P R$, or the turning moment is the tangential pressure times the pitch radius, whatever the obliquity of action and the actual magnitude of the normal pressure. This applies to all systems of gearing. Referring to Fig. 66, it will appear that the reactions at the bearings of the shaft (Q_1, Q_2), hence the load tending to bend the shaft, are dependent on the magnitude of P'. If the distances of the bearings from the gear are b and c (as in Fig. 66),

$$Q_1 + Q_2 = P'\,;\ Q_1 = P' \frac{c}{b+c};\ Q_2 = P' \frac{b}{b+c};$$

$$\therefore \qquad M = Q_1 b = Q_2 c = P' \frac{b\,c}{b+c}. \qquad (4)$$

The bending moment equivalent to the combined bending and twisting action is $M_e = \frac{1}{2} M + \frac{1}{2} \sqrt{M^2 + T^2}$.

The obliquity of the normal pressure at the teeth is thus seen to affect the bending moment on the shaft and the total pressures on the bearings, but it does not affect the twisting moment on the shaft. In cycloidal gearing, the obliquity varies from a maximum at the beginning of the contact, to zero when the contact point lies in the line of centres; and, during the arc of recess, it increases to a maximum at the end of contact. The maximum value of the angle θ (Fig. 65) is about 22° with usual forms of cycloidal gears. When $\theta = 22°$, $\sec \theta = 1.08$, or the maximum normal pressure is about 8 per cent. greater than the tangential, rotative, force. The obliquity is constant throughout the arc of action in involute gears, and the angle θ is usually 14½° or 15°. If $\theta = 15°$, $\sec \theta = 1.035$, or the normal pressure is 3½ per cent. greater than the tangential force.

Mr. Wilfred Lewis, in the *American Machinist* for February 28th, 1901, advocates increasing the obliquity of involute gears to 22½°, to avoid "interference" of the teeth. The secant of this angle is 1.082; and if this angle were adopted, the constant normal pressure would be about equal to the maximum normal

pressure with the cycloidal gears of usual proportions. This would result in somewhat greater journal friction, but the compensating advantage of avoiding interference recommends it for pinions of few teeth and with moderate loads. This greater obliquity would tend to increase the wear on the teeth.

98 **Strength of Gear Teeth.** [Unwin, §§ 210, 211, 218.] The assumptions that the teeth of spur gears can be considered as rectangular prisms in determining their strength is not satisfactory, especially in treating of pinions with a low number of teeth. Fig. 69 shows four gear teeth which have the same thickness at the pitch line, and the same height. The tooth marked (*a*) is one of an involute rack ; (*b*) is one of an involute pinion having 12 teeth ;* (*c*) is one of an epicycloidal gear having 30 teeth ; (*d*) is one of an epicycloidal pinion of 12 teeth.

Mr. Wilfred Lewis, of Wm. Sellers & Co., seems to have been the first to investigate the strength of gear teeth with due regard to the actual forms used in the modern systems of gearing. His work was originally published in the proceedings of the Engineer's Club of Philadelphia, January, 1893. Numerous formulas and diagrams have since been devised for solving the problems connected with the strength of gear teeth.

It is usually intended that more than one pair of teeth shall be in action at all times, but owing to unavoidable inaccuracies of form and spacing; it is not safe to depend upon a distribution of the load between two or more teeth of a gear. It is safest to provide sufficient strength for carrying the entire load on a single tooth. In the rougher classes of work, this load may be concentrated at one edge of the tooth, as indicated in Fig. 67, (see Unwin, § 211). With well supported bearings and machine moulded or cut gears, it is not unreasonable to consider the load as fairly well distributed across the face of the gear, if the face does not exceed about three times the pitch ; see Fig. 69. The obliquity of action gives rise to a crushing action on the teeth (due to the radial component of the normal force), in addition to flexural

*The 12 tooth involute pinion may have its teeth weakened by a correction for interference ; but it is usually better to correct the points of the mating wheel.

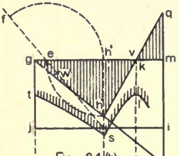


Fig. 64 (b).

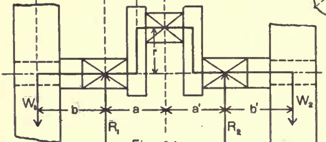


Fig. 64.

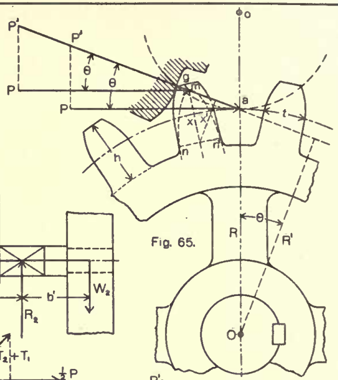


Fig. 65.

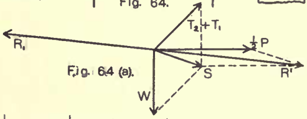


Fig. 64 (a).

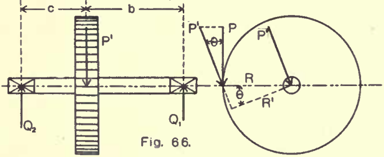


Fig. 66.

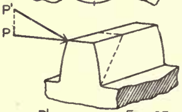


Fig. 67.

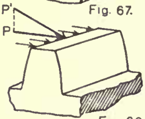


Fig. 68.

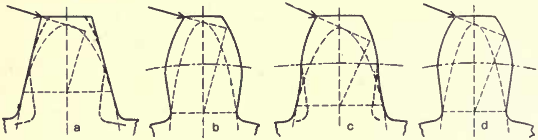


Fig. 69.

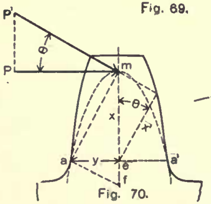


Fig. 70.

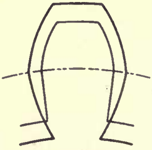


Fig. 72.

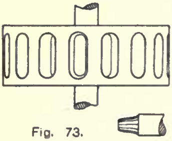


Fig. 73.

stress which results from the tangential force. This crushing component does not exceed about 10 per cent. of the normal pressure. Its effect is to reduce the tensile stress due to flexure, and increases the compressive stress. As cast iron, which is the most common material for gears, is far stronger in compression than in tension, this radial action may usually be neglected.

Referring to Fig. 70, it is seen that the normal force P', when acting on the extreme point of the tooth, produces a bending moment on the cross-section $a\,a'$ equal to $P'\,x' = P\,x$. Let a parabola be drawn with its vertex at m and tangent to the tooth outline curves at a and a'. This parabola represents a cantilever equal in strength to the given gear tooth.

In examining a given form of gear tooth, it is not necessary to actually construct the parabola in order to locate the weakest section with practical accuracy.

The strength of the tooth is given by the following formula :

$$P\,x = \frac{b\,(2\,y)^2 f}{6} = \tfrac{2}{3}\,b\,y^2 f = \tfrac{2}{3}\,b\,p\,f\left(\frac{y^2}{p}\right)$$

in which b is the face of gear teeth, p is the circular pitch, and f is the intensity of stress.

For epicycloidal gears with a diameter of describing circle equal to the radius of a 12-tooth pinion of the same pitch, and fillets equal to the clearance at the root of the teeth, Mr. Lewis gives, as the result of his investigation, the formula,

$$P = b\,p\,f\left(.124 - \frac{.888}{n}\right) \qquad (1)$$

in which P is the load per tooth in pounds, p is the circular pitch in inches, f is the working stress in pounds per square inch, b is the face of the gear in inches, and n is the number of teeth. Mr. Lewis' formula is convenient for determining P, b, p, or f, when the number of teeth (n) is known ; but a common problem in design is to determine the *pitch* when the *pitch diameter* of the gear is given, and the *number of teeth is unknown*. To adapt Mr. Lewis' investigation to this last stated problem, the following is presented, together with a diagram which may be used instead

of numerical computations in solving specific problems. This diagram was published in the *Sibley Journal of Engineering* for June, 1897, and also as a discussion of a paper by Professor F. R. Jones, presented before the A. S. M. E. (Vol. XVIII, page 766).

The first step in the derivation of the new formula is to eliminate the number of teeth (n) and to introduce the pitch diameter (D) in the Lewis expression.

$$p\,n = \pi D \quad \therefore \quad n = \frac{\pi D}{p}$$

$$\therefore \qquad P = b\,p\,f\left(.124 - \frac{.888}{n}\right) = b\,f\left(.124\,p - \frac{.28\,p^2}{D}\right) \qquad (2)$$

If the load *per inch of gear face* is $P_1 = P \div b$,

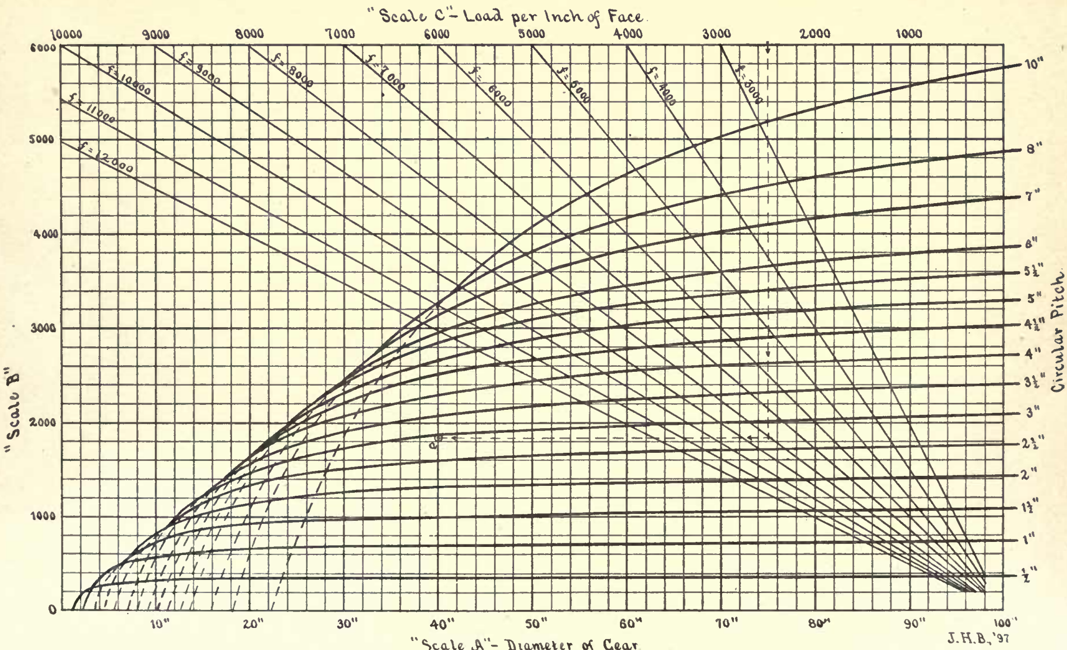
$$P_1 = f\left(.124\,p - \frac{.28\,p^2}{D}\right) \qquad (3)$$

$$\therefore \qquad p = D\left(.22 - \sqrt{.049 - \frac{3.57\,P_1}{f\,D}}\right) \qquad (4)$$

The pitch can be found from eq. (4) for any values of P_1, D, and f, when the face of gear is known or assumed. A common problem is as follows: The distance between two shafts and their velocity ratio is known, required the pitch of spur gears connecting these shafts for a given load and working stress on the teeth. The centre distance of the shafts and the velocity ratio fix the diameters of the gears. The face of the gears may be governed, approximately, by the space available, or it may be assumed by the designer upon other considerations. To illustrate; suppose $P = 15{,}000$ lbs., $f = 8{,}000$ lbs. per sq. inch, and that the smaller gear is to be 40″ diameter. Also, that the face of the gear may be taken as 6″. The load per inch of face is $P_1 = 15{,}000 \div 6 = 2{,}500$ lbs. Hence,

$$p = 40\left(.22 - \sqrt{.049 - \frac{3.57 \times 2{,}500}{8{,}000 \times 40}}\right) = 3.0''.$$

The diagram (Fig. 71) consists of a series of curves (one for each separate pitch), the abscissas of which (scale "A") represent



"Scale C" - Load per Inch of Face
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
f = 10000
f = 9000
f = 8000
f = 7000
f = 6000
f = 5000
f = 4000
f = 3000
f = 11000
f = 12000
6000
5000
4000
3000
2000
1000
0
"Scale B"
10"
8"
7"
6"
5½"
5"
4½"
4"
3½"
3"
2½"
2"
1½"
1"
½"
Circular Pitch
10"
20"
30"
40"
50"
60"
70"
80"
90"
100"
"Scale A" - Diameter of Gear
J.H.B., '97

FIG. 71.

diameters of gears and the ordinates (scale "B"), the load per inch of face, *for a stress of 6,000* lbs. per sq. inch. Any other stress could have been taken for plotting the diagram, and any other stress may be used in solving problems by it.

To illustrate the construction of the diagram for one curve, take that one corresponding to 2″ pitch, and let $f = 6{,}000$. Substituting 2 for p, and 6,000 for f in eq. (3),

$$P_1 = 1{,}488 - \frac{6{,}720}{D}\text{ ; hence, when}$$

$D = 4.5$, $P_1 = 0$; $D = 10$, $P_1 = 816$; $D = 20$, $P_1 = 1{,}152$; etc.

Plotting the corresponding values of D and P_1 as abscissas and ordinates, respectively, the curve for $p = 2''$ is drawn through these points. The other curves are constructed in a similar way.

If any one stress, as 6,000, were proper under all circumstances, a diagram constructed as just explained would be sufficient ; but different materials have different safe working stresses, and the stress for any given material should be reduced as the speed or liability of shock increases. The diagonal stress lines, radiating from the lower right hand corner, are provided for use with other stresses.

The upper horizontal scale ("C") reads from right to left, and its divisions correspond to those of scale "B."

It appears from eq. (2) that the load on the tooth, for any given pitch, varies directly as the stress. Or, from eq. (3), the load per inch of face varies directly as the stress, for any pitch. Hence, for any pitch, when the load per inch of face and the stress are fixed, the given load multiplied by 6,000 and divided by the assigned stress is the equivalent load (for this pitch) with a stress of 6,000 lbs. per sq. inch. Thus, a load of 2,000 lbs. per inch of face and a stress of 3,000 lbs. per sq. inch requires the same pitch as a load of $2{,}000 \times 6{,}000 \div 3{,}000 = 4{,}000$ lbs. at a stress of 6,000. The pitch for a gear of, say, 60″ diameter, load 4,000 lbs. per inch of face, stress equal 6,000, is found, from the diagram, to be about $7\frac{1}{2}''$; which would be the pitch for a unit load of 2,000 lbs. and stress = 3,000. The diagonal lines are so drawn that by

passing vertically downward from any reading (unit load) on scale "C" to one of these diagonals; thence horizontal to scale "B," the load indicated by the reading on "B" will be the equivalent load for a stress of 6,000 lbs. per sq. inch.

To illustrate the use of the diagram, take $P_1 = 2{,}500$, $f = 8{,}000$, $D = 40$. From 2,500 on scale "C," pass vertically downward to the diagonal marked "$f = 8{,}000$"; then horizontally to point "a", on the vertical rising from $D = 40''$ (scale "A"). The nearest pitch curve is that marked $3''$. This curve passes somewhat above the point a, hence the required pitch is somewhat less than $3''$.

If it is required to find the load per inch of face for a gear of given diameter and pitch, with an assigned stress, start at the point on scale "A" corresponding to the diameter; pass upward to the given pitch curve; thence horizontally (right or left, as the case may be) to the appropriate stress diagonal; thence upward to scale "C," where the unit load is read off.

If the diameter, pitch, and unit load are the known quantities, pass upward from the diameter reading on scale "A" to the proper pitch curve; thence horizontally to a point under the unit load reading on scale "C," when the stress is found by interpolation between the adjoining diagonals.

Professor F. R. Jones presented a set of diagrams (in a paper before the A. S. M. E. previously mentioned in this article) which are applicable for similar solutions to those discussed above. He based his work upon a system of gearing in which the diameter of the generating circle is equal to the radius of a 15-tooth pinion, and the fillets are drawn with a "radius equal to one-sixth the space between the teeth at the addendum circle." With this latter system the load per tooth is given by the equation:

$$P = p\,b\,f\left(0.106 - \frac{0.678}{n}\right) \qquad (5)$$

The system used by Mr. Lewis gives teeth that are stronger by from 6 per cent. in a 15-tooth pinion to 17 per cent. in a rack.

Of course, such a diagram as that of Fig. 71 could be drawn for any system of teeth; but the particular system adopted will be

fairly satisfactory for most ordinary cases, with an interchangeable system of epicycloidal gearing.

For use with a limited class of gears, a diagram drawn to a larger scale and covering a smaller range would be better than the one here shown for illustration.

Involute teeth are generally of considerably stronger form than corresponding cycloidal teeth.

It may be noticed that the different pitch curves of Fig. 71 all have a common tangent through the origin o. The points of tangency correspond to the diameters of gear at which the teeth have radial flanks for the respective pitches. It is also seen that the various pitch curves intersect. The intersection of the 10″ and 8″ curves, for example, corresponds to a diameter of about 41″. The interpretation is that a tooth of 8″ pitch (in this system) is as strong as one of 10″ pitch, when the diameter is 41 inches. The reasons for this are · that the teeth of 10″ pitch are longer, and a 41″ gear of 10″ pitch has a number of teeth = 13 —, while an 8″ pitch, with the same diameter, gives a number of teeth = 16 +. For diameters less than 41″, the 8″ pitch is the stronger. See Fig. 72. With a diameter of 22″ +, the 10″ pitch teeth would be so "under-cut" at the flanks that their thickness would be reduced to zero, except for the fillets at the bottom (see scale "A," Fig. 71); while the same condition is not reached with an 8″ pitch until the diameter is reduced to 18″.

An easily remembered relation is: *The load per tooth on a 36-tooth gear equals one-tenth the face of gear in inches multiplied by the circular pitch in inches and stress in pounds per square inch.* A 12-tooth pinion will carry one-half ($\frac{1}{2}$), and a rack one and one-fourth ($1\frac{1}{4}$) the load of a 36-tooth gear with a given stress.

The data may be such that a point (corresponding to "*a*") will lie to the left and above all of the pitch curves of Fig. 71; *i. e.*, above the common tangent through o. If the same data were used in eq. (4), page 158, an imaginary quantity would result. This means that the unit load taken cannot be carried with the stress and diameter assigned, by *any possible pitch.* The only recourse, for a gear of the given diameter and total load (P), is to

increase the face and thus reduce the unit load (P_1), or to use a material which permits a higher intensity of stress.

It is often advantageous, with gears having cast teeth, to "shroud" the smaller gear when the difference in the diameters of a pair of gears is great; see § 218 (Unwin). When the pinion is thus shrouded, its strength may be considered as in excess of that of its unshrouded mate, and the computations for strength can then be applied to the larger gear. Evidently both gears can not be shrouded to the full height of the the teeth; but both may be "half-shrouded," *i. e.*, shrouded to the pitch circle. This latter expedient may be of advantage when the conditions are severe and the gears are of nearly equal diameters.

It is possible to make shrouded "cut" gears by using an "end-mill" for the cutter, as indicated in Fig. 73.

It should be remarked that the teeth of the smaller gear of a pair is subject to the greater wear, as its teeth come into action the more frequently. Hence the teeth of the smaller gear, which are, from their form, initially the weaker, have their strength reduced more rapidly by wear than those of the mating gear.

99. Limiting Velocity of Toothed Wheels. [Unwin, §§ 215, 216]. Small toothed gears are seldom run at such speeds that they are in danger of bursting under centrifugal action; but large fly-wheel gears may approach a dangerous rim velocity. The safe limit is discussed by Professor Unwin in § 215.

100. Strength of Bevel Gear Teeth. [Unwin, § 217]. According to Mr. Lewis, a spur gear of pitch and diameter equal to the pitch and diameter of the bevel gear at the *large ends* of the teeth, is stronger than the bevel gear, in the ratio of D to d; when D is the pitch diameter at the large end, and d the pitch diameter at the small end. This assumption may be made except when the face of the bevel gear teeth is excessively long. Using the notation above, and other notation as in art. 98,

$$P_1 = fp\left(.124 - \frac{.28\,p}{D}\right)\frac{d}{D}; \qquad (1)$$

$$\therefore \quad p = D\left(.222 - \sqrt{.048 - \frac{3.57\,P_1}{fd}}\right) \qquad (2)$$

Compare eqs. (3) and (4) of art. 98.

It is more difficult to insure the uniform distribution of load along the elements of bevel gears than in spur gears. For this reason the length of face should not be unnecessarily long in bevel gear teeth.

101. **Width of Face of Gears.** [Unwin, § 219]. The strength and the durability of gear teeth increase with increase of face, if the shafts are in perfect alignment. The difficulty of securing uniform distribution of the load along the contact element increases as the face becomes greater. This imposes a practical limit to increase of tooth face. The space available for the gears and the "over-hang" (if the gears are on the projecting ends of shafts) sometimes fix the limit of face.

The space available does not usually prevent extending the face of bevel gear teeth in the direction toward the apex of the cone, the large diameter of the gear being determined. However, little is gained, and serious difficulty is encountered by going to an extreme in this respect. The portion of the teeth added by extension toward the intersection of the shafts is of small strength, relatively to a similar length of face near the large ends of the teeth. And the difficulty of securing uniform distribution of load along the teeth elements (mentioned in the preceding article) is increased by such extension. If the bevel gear is cut by the ordinary milling cutter process, the tooth elements do not converge accurately toward the apex of the pitch cone, and this error increases with the length of face. With cast gears, or gears with planed or "moulded" teeth, this last objection does not hold.

102. **Rims of Gears.** [Unwin, § 220]. The thickness of the rim of a gear is commonly about equal to the thickness of the teeth at the roots. This proportion is usually desirable on the score of securing good castings, though it may be departed from. If the distance between arms is so great that more rigidity of rim is desirable (which is generally the case) the rim is ribbed, as indicated in Fig. 236 (Unwin). Small gears are often made solid, that is, of the same thickness from rim to hub. A plate gear is used if the diameter is rather too great for a solid gear, but not

great enough to make the use of arms desirable; that is, a web, or flat plate, extends along the mid-plane of the gear from rim to hub.

103. Arms of Gears. [Unwin, § 221.] In small gears the arms are usually proportioned largely by the judgment of the designer. The thickness of metal in the arms should not differ greatly from the adjacent thickness at the rim and at the hub, for if the casting does not cool quite uniformly the shrinkage stresses often exceed those due to the load transmitted. Professor Unwin gives three methods of computing the arms. The first, in which the ratio $h \div a$ is assumed, may be used, but it is perhaps better in the usual case to make the thickness a and β (Fig. 241, Unwin) about equal to the rim thickness, or about equal to ½ the pitch. If a be taken as $.5\,p$ in eq. (14),

$$h^2 = \frac{12\,PR}{v\,f\,p} \quad \therefore \quad h = \sqrt{\frac{12\,PR}{v\,f\,p}}, \qquad (1)$$

The second method given in Unwin is based upon the assumption that the gear tooth can be treated as a rectangular prism in considering its strength, which is not in accordance with the preceding work on strength of gear teeth.

The third method given by Unwin seems best for the usual case, but eq. (1), above, may be used instead of eq. (18) of Unwin. The text of § 221 should be read; but eqs. (16), (17) and (18) may be omitted.

104. Hubs of Gears. [Unwin, § 222]. It is quite common to make the diameter of the hub twice the diameter of the shaft; that is, the thickness of metal in the hub is equal to the radius of the shaft. In case of a light gear on a large shaft, this rule would give excessive thickness of metal; that is, more than strength demands, and also a difference between hub and rim thicknesses which would tend to unnecessarily increase the shrinkage stresses. For such conditions as these, the hub should be lighter than the common rule indicates.

It is not usual, in this country, to enlarge the shaft where it passes through the gear; the preference being for a straight shaft

large enough to permit keyseating without undue weakening of the shaft. This practice usually saves more in shop work than the opposite course would save in material ; though there are exceptions to this rule.

The term "nave" is the British name for what is usually called the hub in this country.

105. **Weight of Toothed Gearing.** [Unwin, § 223].

106. **Effiicency of Spur Gearing.** The experimental data on the efficiency of spur gears is apparently very meagre. Probably the best available data are those obtained by Mr. Wilfred Lewis, for details of which see Trans. A. S. M. E. Vol. VII, page 273. His investigation was made with a cut spur pinion of 12 teeth meshing with a gear of 39 teeth. The pitch was 1½″ and the face was 3⅜″. The load was varied from 430 lbs. to 2,500 lbs. per tooth, and the peripheral speed ranged from 3 feet to 200 feet per minute. The measurements included the friction at the teeth and the friction of the two shafts. The efficiency, as observed, varied from 90 per cent. at a velocity of 3 feet per minute to over 98 per cent. at 300 feet per minute. It appears that the friction at the teeth is a small part of the loss, with good cut gears ; the greater portion of the loss being at the journals. This latter is, however, a necessary loss incident to the use of gearing.

107. **Helical or Twisted Gearing.** [Unwin, §§ 224 to 226].

108. **Screw Gearing.** [Unwin, §§ 227–228]. The expression for the efficiency (η) of screw gears [eq. (26), Unwin] can be reduced to the following :

$$\eta = \frac{\tan\theta\ (1 - \mu \tan\theta)}{\tan\theta + \mu} \qquad (1)$$

in which θ is the inclination of the pitch line helix to a plane perpendicular to the axis ; and μ is the coefficient of friction. This does not include the friction at the thrust bearing, which is often an important item in a worm and wheel mechanism. The

following is an approximate expression for the efficiency, including the thrust bearing.*

$$\eta = \frac{\tan \theta (1 - \mu \tan \theta)}{\tan \theta + 2 \mu} \qquad (2)$$

The above formulas are based upon the assumption that the worm teeth correspond to the threads of a square threaded screw. As this is not the case, it would be natural to expect the real efficiency to be somewhat lower than these expressions give. The experiments of Mr. Lewis show a very satisfactory agreement with the latter formula. See Trans. A. S. M. E. Vol. VII, p. 273; also article by Mr. F. A. Halsey, *American Machinist* for Jan. 13th and 20th, 1898. An abstract of these last named articles will be found in Kent's Mechanical Engineer's Pocket-Book, page 1078.

The frictional work at the teeth of a spiral gear is proportional to the velocity of rubbing; hence the efficiency increases as the diameter decreases, for a given rotative speed. An examination of eqs. (2) or (3) shows that the efficiency increases very rapidly with increase of the inclination (θ) at low angles. The maximum efficiency if $\mu = .05$ is at an angle of about $43\frac{1}{2}°$ (neglecting the thrust bearing) or at about 53° (including the thrust bearing). With an increase of the angle beyond these limits, the efficiency falls off. The smaller inclinations correspond to single threaded worms, or at least to worms of only a few threads. The higher angles are obtained with spiral gears of several threads. With a value of θ much greater than 45°, the other gear of the pair approaches more nearly to the special form of spiral gear commonly called a worm, because the inclinations of the helices of the pair are complimentary when, as is most common, the shafts are at right angles.

If $\theta = 60°$ for one of the gears, the inclination of the helix of the mating gear would be 30°, if the axes are at right angles. It

* Equation (2) is based on these assumptions: that the mean diameter of the thrust collar is equal to the pitch diameter of the worm; and that the coefficient of friction is the same at the teeth and thrust collar.

would therefore be reasonable to expect about the same efficiency at $\theta = 30°$ and $\theta = 60°$. Neglecting the friction at the thrust bearing, this would be substantially the result.

A ball bearing step has been used to good purpose in reducing the step friction. The objection to this device is the danger of cutting the ball races under heavy loads.

109. **Construction of Screw Gearing.** [Unwin, §§ 230 to 236.] The usual method of making accurate worm wheels is to use a "hob", which is a milling cutter of similar general form to the worm which is to mesh with the wheel. See Kinematics of Machinery, page 168. For this reason it is seldom necessary to make an accurate drawing of a worm. However, worms with cast threads are still used in some classes of heavy, rough work, and the method of laying out the teeth is fully treated in §§ 233 to 236 (Unwin).

110. **Strength of Worm Wheels.** [Unwin, § 236.]

X.

BELT TRANSMISSION.

111. **Materials of Belts.** [Unwin, page 369.]

112. **Velocity Ratio in Belt Transmission.** [Unwin, § 237.]

113. **Resistance to Slipping of Belts.** [Unwin, §§ 241, 242.] The equations given in § 241 are very generally used; but the assumptions on which they are based are only approximations to the conditions of operation. This theory considers slipping as *impending*. It is probable that slippage occurs whenever the belt transmits power, and that the rate of slippage gradually increases with the effective, or unbalanced, belt pull, to a certain point at which the belt either runs off the pulleys or slips so much that it fails to drive. The coefficient of friction is not constant, but it increases with slippage.

114. **Tensions in an Endless Belt.** [Unwin, §§ 243, 244.] The assumption that the sum of the tensions on the two sides of the belt remains constantly equal to the sums of the initial tensions has been proved to be in error by the experiments of Messrs. Lewis and Bancroft, (Trans. A. S. M. E., Vol. VII, page 549). The equations given by Professor Unwin are, therefore, not exact; though they are convenient for many computations, and are those which have generally been accepted. A brief abstract of Mr. Lewis's paper is given in art. 122, below.

115. **Strength of Leather Belting.** [Unwin, § 245.] The practice of Wm. Sellers & Co., as reported by Mr. Lewis in the transactions of the A. S. M. E., Vol. XX, page 152, is to take $f = 400\,\delta$ for cemented belts, (with no laced joints); and $f = 275\,\delta$ for laced belts. In this relation, f is the tension on the tight side of the belt in lbs. per inch of width, and δ is the thickness of the belt in inches; as in § 245 (Unwin). A still lower stress increases the life of the belt.

116. **Width of Belt for a given Stress.** [Unwin, § 246.]

117. **Horse-power per inch of Belt Width.** [Unwin, § 247.]

118. **Rough Calculations of Belt Width.** [Unwin, § 248.]

119. **High Speed Belting.** [Unwin, § 249.] It appears that the effect of a high speed of belt frequently tends to reduce the adhesion, rather than to increase it by formation of a partial vaccuum. There are two causes for this reduction of adhesion. One of these, the centrifugal action due to the weight of the belt, will be treated later ; the other cause is the adhesion of air to the belt. This latter action tends to carry a film of air between the belt and the pulley, as the oil film is carried into the space between a journal and its bearing. The viscosity of oil is much greater than that of air ; but, on the other hand, the velocity of the belt is very high, compared with that of a journal. To allow this entrained air to escape, belts are sometimes perforated, usually with oblong holes in order to avoid excessive weakening of the belt. This practice tends to increase the stretch of the belt. Occasionally the pulley rim is perforated to allow the escape of the air without reduction of the cross-section of the belt.

120. **Influence of Elasticity of the Belt.** [Unwin, § 250.] The slippage of belts, as measured in belt tests, is made up of creeping due to the stretch of the belt, and real slippage of the belt on the pulleys ; both of which occur when power is transmitted.

121. **Effect of Centrifugal Action.** [Unwin, § 251.]

122. **Recent Investigations of Belt Transmission.** A number of important investigations of belt transmission have been reported to the American Society of Mechanical Engineers. See the following papers in the transactions of the Society, by : Mr. A. F. Nagle, Vol. II, page 91. Professor G. Lanza, Vol. VII, page 347. Mr. Wilfred Lewis, Vol. VII, page 549. Mr. F. M. Taylor, Vol. XV, page 204. Professor W. S. Aldrich, Vol. XX, page 136. Abstracts of some of these papers, as well as other valuable data, are given in Kent's Mechanical Engineers' Pocket-Book, pages 876 to 887.

Mr. Lewis' paper gives the results of and conclusions from the very careful tests conducted by himself and Mr. Bancroft for William Sellers & Co. The apparatus used by them was afterward presented to Sibley College, and is now used by the Department of Experimental Engineering. These tests indicate that with open or straight belts, the journal friction is the principal resistance at moderate speeds, and that air resistance becomes appreciable at high speeds. With crossed belts, the rubbing together of the sides of the belt in crossing, and the resistance at the point where the belt leaves the pulley are often sources of considerable loss. The bending of the belt around the pulleys did not result in an appreciable loss of energy ; narrow thick belts being as efficient as wide thin belts, apparently. The rate of the strain in leather *decreases* with the stress, instead of increasing as in the case of ductile metals. This property is similar to that possessed by soft rubber, and it becomes very apparent in the latter material when a common rubber band is stretched out by the fingers. As a result of this property of leather, the sum of the tensions on the two sides of the belt is not constantly equal to the sum of the initial tensions. The reason for this is as follows: When the belt transmits power, the tension is increased on the driving side and is decreased on the slack side. A given reduction of tension on the slack side tends to shorten that side of the belt more than the tight side is increased in length by the same increase of its tension ; consequently the resultant effect of transmission of power by a belt tends to shorten its length as a whole or to increase the sum of the tensions. Suggestion :—Place a rubber band over the fingers of the two hands and stretch it moderately ; then twist one of the hands in either direction and the increase of force tending to bring the hands together will be apparent.

In case of a long horizontal belt, the increase in the sum of the tensions is still further augmented in driving, because the tension on the slack side (with a proper initial tension on the belt) is largely due to the sag of the belt from its own weight, and thus the tension on the slack side tends to remain nearly constant,

while the tension on the tight side increases with the power transmitted, at a given belt speed. It appears that the sum of the tensions on the two sides when driving may exceed the sum of the initial tensions by about 33 per cent. in vertical belts, and, in horizontal belts, the increase may be limited only by the strength of the belt. In addition to the causes just discussed, the tensions in both sides of the belt are increased by the centrifugal action due to the mass of the belt. This latter cause increases the stress in the belt and decreases adhesion between the belt and the pulley, but it does not increase the loads on the shafts which produce pressure at the bearings and flexure of the shafts.

The slippage of the belt, under good conditions, may be assumed at about 2 per cent.; about 1 per cent. being "creeping" due to the elasticity of the material, and the remainder being true slip, or sliding of the belts on the pulleys.

The coefficient of friction increases with the velocity of sliding, up to the point at which the belt tends to leave the pulley. A given actual velocity of sliding represents a smaller percentage of slip as the belt speed increases; hence the actual velocity of sliding and the coefficient of friction may be increased at higher speeds, while the percentage of slippage is reduced. The slippage may become 20 per cent. or more before the belt leaves a crowned pulley; but much lower slippage is desirable on the score of efficiency and durability of the belt. If the tension on the slack side is too low, the slippage becomes excessive; on the other hand, too great tension on the belt results in unnecessary increase of the journal friction, and excessive wear of the belt.

Either of these extremes reduces the efficiency of transmission. It appears that, with favorable conditions, the efficiency of transmission by an open belt may be as high as 97 per cent.

The coefficient of friction of the belt on the pulley may be taken at about .40, except for dry belts at slow speeds. With an arc of contact of 180°, the coefficient of friction was found to be about:

$\mu = .25$ for dry oak tanned leather at a speed of 90 feet per minute, and

$\mu = 1.38$ for a very flexible rawhide belt at 800 feet per minute.

A value of $\mu = 1.00$ is quite possible, though not to be depended upon for ordinary working conditions.

Mr. Nagle gives the following formula for the power transmitted, having due regard to the effect of centrifugal action in increasing the tension :

$$H.P. = C\,V\beta\,\delta\,(f - .012\,V^2) \div 550 \qquad (1)$$

in which $V =$ the velocity of the belt in feet per minute, $\beta =$ the width of the belt in inches, $\delta =$ the thickness of the belt in inches, $f =$ the working strength of leather in lbs. per sq. inch cross-section, and $C =$ a constant determined by the formula,

$$C = 1 - 10^{-.00758\,\mu\,\theta} \qquad (2)$$

when $\mu =$ the coefficient of friction, and $\theta =$ the arc of contact in degrees.

Solving eq. (1) for the width of belt,

$$\beta = \frac{550\,H.P.}{C\,V\,\delta\,(f - .012\,V^2)} \qquad (3)$$

The power transmitted by a belt increases directly with the belt velocity, except for the effect of the contrifugal action. The stress due to centrifugal action increases with the square of the velocity ; hence, for a given value of the working stress (f), there is a limiting velocity at which the greatest power is transmitted. Differentiating eq (1) with reference to $H.P.$ and V, considering the other quantities as constant, placing the differential coefficient equal to zero, and solving for V, it will be found that

$$V = \sqrt{28f} = 5.29\sqrt{f} \qquad (4)$$

If f be taken at 400 lbs. per sq. inch of section for cemented belts (see art. 115), $V = 5.29 \times 20 = 105.8$ feet per second, or 6350 feet per minute. If f is taken at 275 lbs. per sq. inch for laced belts, $V = 5.29 \times 16.6 = 87.8$ feet per second, or 5270 feet per minute. It is often necessary to run belts at much lower speeds than these ; but it is not economical to exceed these limits. A belt speed of a mile a minute may be taken as about the

economical limit ; and it so happens that this is also about the limit of safety for ordinary cast iron pulley rims. See § 259, (Unwin). For durability combined with efficiency, a speed of 3000 to 4000 feet per minute may be taken as a fair belt speed. It was stated above, that the coefficient of friction may vary from .25 to 1.00 ; a fair general value being about .40. In this connection, Mr. Lewis says : "This extreme variation in the coefficient of friction does not give rise, as might at first be supposed, to such a great difference in the transmission of power. It will be seen by reference to formula (1) that the power transmitted for any given working strength and speed is limited only by the value of C, which depends upon the arc of contact and the coefficient of friction. For the usual arc of contact 180°, the power transmitted when $\mu = .25$ is about 24 per cent. less than when $\mu = .40$ and when $\mu = 1.00$, the power transmitted is about 33 per cent. more, from which it appears that in extreme cases the power transmitted may be $\frac{1}{4}$ less or $\frac{1}{3}$ more than will be found from the use of Mr. Nagle's coefficient of .40."

The paper by Mr. Taylor, referred to above, gives an account of "A Nine Years' Experiment on Belting," that is a record of careful observations and measurements for nine years on belts in actual use at the works of the Midvale Steel Co Many valuable facts and practical suggestions are contained in this paper, but a satisfactory abstract of it is not possible in this place. Mr. Taylor advocates thick narrow belts, rather than thin wide belts. He sums up his investigation in 36 "Conclusions" ; the first of which is that :

"A double belt having an arc of contact of 180°, will give an effective pull on the face of the pulley per inch of width of belt of" 35 lbs. for oak tanned and fulled leather belts, or 30 lbs. for other types of leather belts and 6 to 7 ply rubber belts.

"The number of square feet of double belt passing around a pulley per minute required to transmit one horse-power is" 80 sq. feet for the oak tanned belt, or 90 sq. feet for other leather belts and 6 to 7 ply rubber belts.

"The number of lineal feet of double belting 1 inch wide passing around a pulley per minute required to transmit one horse-

power is" 950 feet for the oak tanned belt; or 1,100 feet for the other types as above.

These conclusions are based upon the cost of maintaining the belts in good condition, including loss of time from repairs, as well as other considerations. Smaller values than these rules dictate are generally used, because of the first cost their application would involve.

123. **Joints in Belting.** [Unwin, § 252]. Belts are not infrequently made without any laced joint. In belted dynamos, the belt is tightened by moving the machine on its bed plate; suitable adjusting screws being provided for the purpose. In other cases, a tightening pulley is provided; and, again, the belt may be cemented by a scarf joint, and a new joint be made if it becomes necessary to tighten the belt. Small belts usually have a laced joint. If a tightener is used it should run against the slack side of the belt, and it is usually best to place it near the smaller pulley to increase its arc of contact rather than that of the larger pulley.

124. **Cotton Belting.** [Unwin, § 253]. Cotton belting with a thin leather contact facing has been used to a considerable extent in this country.

125. **Leather Link Belting.** [Unwin, § 254].

126. **Proportions of Pulleys.** [Unwin, §§ 258 to 263, inclusive].

XI.

ROPE TRANSMISSION.

127. **Transmission Ropes and Sheaves.** [Unwin, Page 404 to § 265, and also § 269].

128. **Strength of Ropes**. [Unwin, § 265]. The working stress of 1200 lbs per square inch for hemp ropes, as assigned by Professor Unwin, seems to be much in excess of that found desirable by experience. Durability of ropes demands a much smaller working stress, relatively to the ultimate strength, than would be provided by a factor of safety of 8 in a new rope.

Mr. C. W. Hunt, Past President of the American Society of Mechanical Engineers, presented the conclusions reached from his extensive experience with rope transmission in a paper before the Society, (Transactions Vol. XII, page 230). An abstract of this paper is given in art. 131 of these Notes.

The student is referred to the work in Unwin's Machine Design for the general theory of rope transmission; but the formulas and data given in Mr. Hunt's paper are recommended for use in applications.

129. **Driving Force and Power Transmitted by Ropes.** [Unwin, §§ 266, 267].

130. **Friction of Ropes in Grooved Sheaves.** [Unwin, § 268].

131. **Manilla Rope Transmission.** The following is, in the main, an abstract of Mr. Hunt's paper before the A. S. M. E., to which reference was made in art. 128; the quotations being his own words: "The most prominent questions which the engineer wishes to have answered who proposes to make an application of rope driving are those relating to—

Horse-power,
Wear of rope,
First cost of rope,
Catenary.

These questions cannot be answered with precision in a general article, but it is the purpose of this paper to give the general limitations of this method of transmitting energy."

"In many of the earlier applications so great a strain was put upon the rope that the wear was rapid, and success only came when the work required of the rope was greatly reduced. The strain upon the rope has been decreased until it is approximately known what it should be to secure reasonable durability." Experience indicates that a tension in the driving side of the rope equivalent to 200 lbs. on a manila rope 1 inch in diameter is safe and economical.

Tests of ropes from different makers showed an average breaking strength equivalent to 7,140 pounds on a rope one inch in diameter.

The following notation, used throughout this article, has been changed to correspond to that used by Professor Unwin.

$\gamma =$ Circumference of rope in inches.

$\delta =$ Sag of rope in inches.

$C =$ Centrifugal force in pounds.

$H.\ P. =$ Horse-power transmitted.

$L =$ Distance between pulleys in feet.

$w =$ Weight of rope in lbs. per foot of length.

$P =$ Effective force (or tension) in the rope in lbs.

$T_2 =$ Tension on driving side of rope in lbs.

$T_1 =$ " " slack " " " " "

$v =$ Velocity of rope in feet per second.

$f =$ Working stress in one rope.

$F =$ Breaking strength of one rope.

According to Mr. Hunt:

$$F = 720\gamma^2 \qquad (1)$$

$$f = 20\gamma^2 \qquad (2)$$

$$w = .032\gamma^2 \qquad (3)$$

Since $F = 36f$; the apparent factor of safety is 36. The working stress is about one twenty-fifth the effective strength of a new rope, allowing for the splice. "The actual strains are ordinarily much greater, owing to the vibrations in running, as well as from imperfectly adjusted tension mechanism."

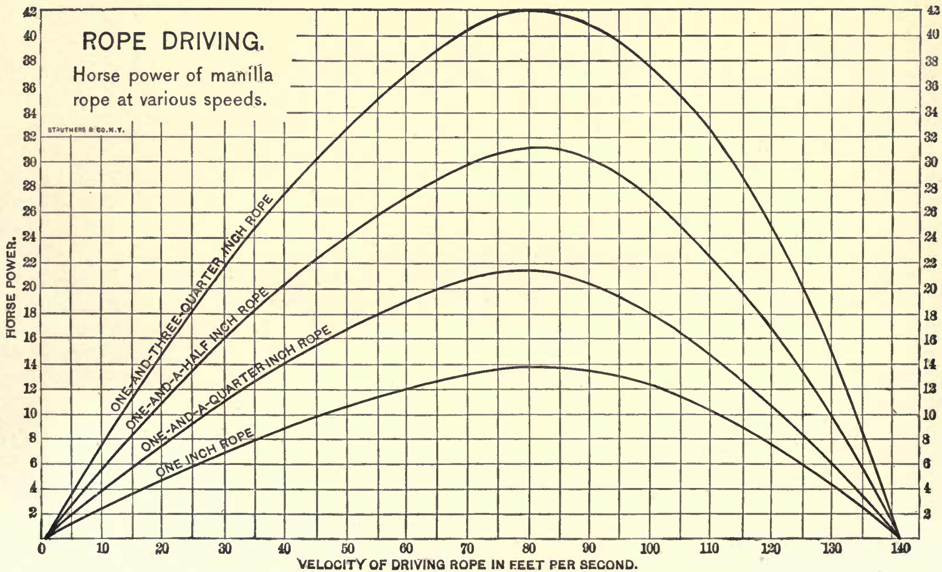

ROPE DRIVING.
Horse power of manilla rope at various speeds.
STRUTHERS & CO. N.Y.
HORSE POWER.
VELOCITY OF DRIVING ROPE IN FEET PER SECOND.
ONE-AND-THREE-QUARTER-INCH ROPE
ONE-AND-A-HALF INCH ROPE
ONE-AND-A-QUARTER-INCH ROPE
ONE INCH ROPE

FIG. 74.

The maximum working stress per rope is taken as equivalent to 200 lbs. on the driving side of a 1 inch rope ($f = 20\gamma^2$); and the range of velocity is from 10 to 140 feet per second.

The centrifugal action produces a tension of

$$C = \frac{w v^2}{g} = \frac{.032\,\gamma^2 v^2}{g} = \frac{\gamma^2 v^2}{1000} \text{ (nearly)} \qquad (4)$$

If $v = 141$, $C = 20\gamma^2 = f$, hence the centrifugal tension equals the allowed working stress, and the effective pull becomes zero; that is no power can be transmitted at this speed without exceeding the assigned maximum tension in the rope.

Mr. Hunt states that there have been no experiments which accutately determine the coefficient of friction of lubricated ropes on pulleys; but that "when a rope runs in a groove whose sides are inclined toward each other at an angle of 45° there is sufficient adhesion when"

$$T_2 \div T_1 = 2 \qquad (5)$$

However, he assumes a somewhat different ratio of T_2 to T_1 in the following work, viz: "That the tension on the slack side necessary for giving adhesion is equal to one-half the force doing useful work on the driving side of the rope." Both sides of the rope have a component of tension due to the centrifugal action $= C$. If the tension for adhesion be called t_0, $T_1 = t_0 + C$, and $T_2 = P + t_0 + C$; but if t_0 be taken as $\frac{1}{2} P$, $T_1 = \frac{1}{2} \cdot P + C$, and $T_2 = P + \frac{1}{2} P + C = \frac{3}{2} P + C$. From these relations,

$$P = \frac{2(T_2 \quad C)}{3} \qquad (6$$

$$t_0 = \frac{1}{2} P = \frac{T_2 - C}{3} \qquad (7)$$

$$T_1 = t_0 + C = \frac{1}{2} P + C = \frac{T_2 - C}{3} + C \qquad (8)$$

From the above relations,

$$\frac{T_2}{T_1} = \left(\frac{3}{2} P + C\right) \div \left(\frac{1}{2} P + C\right) = \frac{3P + 2C}{P + 2C}$$

that is, the ratio of the total tensions on the two sides of the rope varies with the effective pull and with the speed of rope. With the assumption that the tension for adhesion is one-half the effective pull ($t_0 = \frac{1}{2} P$), $T_2 \div T_1$ equals 2 when $P = 2C$. This

corresponds to a velocity of 70.7 feet per second. With a velocity of 80 feet per second, which is about the velocity for maximum power transmitted, $T_2 \div T_1 = 1.83$.

It follows from eq. (8) that $(T_2 + T_1) = (4\,T_1 - 2\,C)$; or that the sum of the tensions is not constant for all speeds. This last conclusion is arrived at without reference to such peculiar relations between stress and strain as those discussed in art. 122. It is probable that ropes, like belts, undergo decreasing increments of strain with increasing increments of stress, and that the expression for $(T_2 + T_1)$ should be modified accordingly.

"As C varies as the square of the velocity, there is, with an increasing speed of rope, a decreasing useful force, and an increasing total tension, T_1, on the slack side," for a fixed value of T_2.

With the preceding assumptions, the horse-power transmitted will be:

$$H.P. = \frac{P\,V}{550} = \frac{2\,V(T_2 - C)}{3 \times 550} = \frac{V(T_2 - C)}{825} \qquad (9)$$

Transmission ropes are usually from one to one and three-quarters inches in diameter."

Fig. 74 is a diagram showing the horse-power transmitted by four common sizes of rope, based upon a total tension on the driving side equivalent to 200 lbs. on a one inch rope; or $T_2 = 20\gamma^2$. The electrotype for this diagram was kindly furnished by The C. W. Hunt Company.

If the value of C as per eq. (4) be substituted in eq. (9) and the resulting expression be solved for a maximum, it will be found that the greatest horse-power is transmitted at a velocity of about 82 feet per second. An inspection of Fig. 74 shows its agreement with this result. In order that the value of T_2 shall be maintained equal to $20\gamma^2$, the effective pull must be reduced as the centrifugal force is increased. The energy transmitted equals Pv, and if v exceeds 80 feet per second P must be reduced at a greater rate than v increases, on account of the rate at which the centrifugal force is augmented.

If T_2 is increased with the speed, greater power may, of course, be transmitted at a higher velocity, and the first cost is reduced;

but this is done at the expense of life of the rope. With a fixed value of $T_2 = 20 \gamma^2$, the first cost is a minimum at about $v = 80$, and this first cost is greater by about 10 per cent. if v is increased to 100 or decreased to 62 feet per second. The first cost is increased by 50 per cent. when the velocity is reduced to 40 feet per second.

"The wear of rope is both internal and external; the internal is caused by the movement of the fibres on each other, under pressure in bending over the sheaves, and the external is caused by the slipping and wedging in the grooves of the pulley. Both of these causes of wear are, within the limits of ordinary practice, assumed to be directly proportional to the speed."

Equation (9) shows that the power transmitted does not vary at this same rate. "The higher the speed, up to about 80 feet per second, the more power will be transmitted, but it is accompanied by more than equivalent wear."

The smallest sheave over which a rope runs in a transmission should have a diameter not less than about 40 times the diameter of the rope.

"There are two methods of putting ropes on the pulleys; one in which the ropes are single and spliced on, being made very taut at first, and less so as the rope lengthens, stretching until it slips, when it is respliced. The other method is to wind a single rope over the pulley as many times as is needed to obtain the necessary horse-power and put a tension pulley to give the necessary adhesion and also to take up the wear [stretch]." This pulley is also necessary to convey the rope to the main pulleys in the proper planes.

The catenary or sag of the tight side of the rope is constant at all speeds if the tension on this side is constant. "The deflection of the rope between the pulleys on the slack side varies with each change of the load or change of speed, as the tension equation (8) indicates."

The following formula may be used for computing the approximate sag in inches, taking T as the tension on the side of the rope under consideration; that is, $T = T_2$ for the sag of the tight side, or $T = T_1$ (as given by eq. 8) for the sag of the slack side,

$$\delta = \frac{T}{2w} - \sqrt{\frac{T^2}{4w^2} - \frac{L^2}{8}} \qquad (10)$$

Great care is necessary in the construction of a rope sheave to have all of the grooves of same depth and form. Neglect of this point will result in excessive tension on some of the ropes, while others are subjected to much lower tension.

The pitch, or effective, diameter of a rope sheave is the diameter measured to the center-line of the rope, or to the neutral axis of the rope which wraps around the wheel. The effective diameter is equal to the length of rope required to reach once around the wheel divided by π; this length of rope being taken when it is under the working stress. If two parallel ropes connect two sheaves, imagine the grooves of the driven wheel to be exactly alike, but that one of the grooves of the driver has a larger effective diameter than the other. The linear velocity of the rope running to the groove of larger pitch diameter will be greater than that of the parallel rope; hence it will tend to carry all of the load, with a corresponding increase of its tension and diminution of the tension on the other rope.

132. **Wire Rope Transmission.** [Unwin, §§270 to 277]. The recent development of systems of electrical transmission of power has greatly curtailed the field of wire rope transmission; though there are conditions under which wire rope may still be advantageously employed.

The table on the following page, which is taken from a circular of the John A. Roebling's Sons Co., shows the power that may be transmitted by ropes of various sizes with sheaves of different diameters and rotative speeds. These values are for a rope made with six strands around a hemp core, each strand consisting of seven wires. This table does not make allowances for the change of stress due to change of centrifugal force at various speeds; but it does consider the influence of the sheave diameter on the bending stress. For example: a 5/8″ rope on an eight foot sheave running 100 r. p. m. transmits only 32 H. P.; while the same rope transmits 64 H. P. when running on a ten foot sheave at 80 revs. per minute, or at the same linear velocity.

TABLE OF TRANSMISSION OF POWER BY WIRE ROPES.*

Diameter of wheel in feet.	Number of revolutions	Trade Number of rope.	Diameter of rope.	Horse-Power	Diameter of wheel in feet.	Number of revolutions.	Trade Number of rope.	Diameter of rope.	Horse-Power.
3	80	23	3/8	3	7	140	20	9/16	35
3	100	23	3/8	3 1/2	8	80	19	5/8	26
3	120	23	3/8	4	8	100	19	5/8	32
3	140	23	3/8	4 1/2	8	120	19	5/8	39
4	80	23	3/8	4	8	140	19	5/8	45
4	100	23	3/8	5	9	80	20 19	9/16 5/8	47 48
4	120	23	3/8	6	9	100	20 19	9/16 5/8	58 60
4	140	23	3/8	7	9	120	20 19	9/16 5/8	69 73
5	80	22	7/16	9	9	140	20 19	9/16 5/8	82 84
5	100	22	7/16	11	10	80	19 18	5/8 11/16	64 68
5	120	22	7/16	13	10	100	19 18	5/8 11/16	80 85
5	140	22	7/16	15	10	120	19 18	5/8 11/16	96 102
6	80	21	1/2	14	10	140	19 18	5/8 11/16	112 119
6	100	21	1/2	17	12	80	18 17	11/16 3/4	93 99
6	120	21	1/2	20	12	100	18 17	11/16 3/4	116 124
6	140	21	1/2	23	12	120	18 17	11/16 3/4	140 149
7	80	20	9/16	20	12	120	16	7/8	173
7	100	20	9/16	25	14	80	8 7	1 1 1/8	141 148
7	120	20	9/16	30	14	100	8 7	1 1 1/8	176 185

* Taken from a publication of the John A. Roebling's Sons Company, of Trenton, N. J.

The above table gives the power produced by Patent Rubber-lined Wheels and Wire Belt Ropes, at various speeds.

Horse-powers given in this table are calculated with a liberal margin for any temporary increase of work.

For hoisting, and for transmission if the sheave diameters must be much smaller than those given in the preceding table, a more flexible rope is used. This consists of six strands around a hemp core, but each strand is made up of nineteen wires, which are, of course, of smaller diameter than those used for corresponding sizes of seven-wire rope. The lining of the bottoms of the grooves in the sheaves should be maintained in good repair. If it becomes irregular, through wear, the rope may be bent at a sharp angle in passing over the high spots of the lining, with a resultant increase in the stress of the wires. This last action is not equivalent to running over a correspondingly smaller sheave, however, for every portion of each wire is bent around each sheave once during every circuit of the rope; while it is not likely that the same portion of the rope will frequently come in contact with any irregularity in the lining.

133. **The Catenary and Sag of Rope.** [Unwin, § 285.] The approximate equations of § 285 (Unwin) are sufficiently exact for most problems of practice, and the discussion of §§ 278 to 285 will be omitted.

134. **Efficiency of Wire Rope Transmission.** [Unwin, § 286.]

135. **Pulleys for Wire Rope Transmission.** [Unwin, § 287.]

136. **Velocity, Wear, Stretch, etc.** [Unwin, §§ 288 to 290, inclusive.]

XII.

CHAINS AND CHAIN WHEELS.

137. **Chains used for Cranes, etc.** [Unwin, §§ 291 to 299, inclusive.]

138. **Chains and Sprocket Wheels for Transmission of Power.** [Unwin, §§ 300 to 304.]

See also Kinematics of Machinery, art. 121, pages 214–215.